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AN EVALUATION OF TRAVEL TIME FACTORS

A THESIS

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by

Norman Joseph Ashford

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SUMMARY

In the past decade, considerable experience has been gained in the field of transportation planning with the gravity model, a method of distributing intra-areal trips. The effect of distance upon the probability of travel is expressed in this model by the travel time factor curve. Current practice treats this travel time curve as an empirical hand drawn curve, adjusted by successive iterations of the calibration procedure until the gravity model adequately models the origin-destination study. It is also assumed that the travel time curve remains constant from the calibration year to the design year.

For some time, it has been realized that only by the use of a mathematical curve can the assumption of a constant travel time curve be truly tested, both by examination of change of curve parameters, and by relation of curve parameters to the characteristics of different urban areas. In 1965, R. E. Whitmore in a study entitled "Graphical and Mathematical Investigation of the Differences in Travel-time Factors for the Gravity Model Trip Distribution Formula in Several Specific Urban Areas," examined the nature of travel time factors for various cities and areas. Whitmore concluded that the travel time factor could be represented by a polynomial form, and that the regression coefficients of the polynomial were relatable to area wide variables.

This thesis seeks to extend the work of Whitmore by fitting a more basic differential equation form to the travel time curves, and

developing from this basic equation the Pearson system of curves to enable a parametric fit to existing curves. The travel time factors of ten cities, with a wide geographic and population variation were examined for three different trip purposes. It was found that home based work trips and non home based trips could be modelled with a Pearson Type I distribution, while shopping trips appeared to fit to the Pearson Type III curve.

In all cases the parameters of the modelled curves were statistically relatable by regression equations to area wide variables.

The home based work trip curves were found to be related to:

1. Home based work trips per thousand population.
2. Total number of home based work trips.
3. Total number of trips per car.

The non home based trip curves were found to be related to:

1. Total number of trips per car.
2. Ratio of non home based trips to all trips.
3. Total number of cars.
4. Non home based trips per car.
5. Ratio of non home based trips to size of study area.

The findings with respect to shopping trips indicated that while the Pearson III distribution could be used to model the shopping trip, the correlation with area wide variables left much to be desired, mainly because the sample size was small.

It was found that statistical relations could be determined between curves and the following area wide characteristics:

1. Total trips.

2. Total trips per thousand population.

3. Ratio of home based other than work trips to total trips.

4. Cars per person.

Based on the derived regression relationships, crude estimates of travel time curves could be calculated for some of the cities examined. These crude estimates could be refined using the modelling technique outlined in this work.

CHAPTER I

INTRODUCTION

Since the beginning of the present century, the United States has changed from a rural to an urban society. In 1900, one country only, Great Britain, was such an urbanized state. In the last 50 years, most of the industrialized nations of the world have become predominantly urban rather than rural in structure. The trend has been especially rapid in the United States, which has transformed from a 40 per cent urban society in the early nineteen hundreds to 70 per cent urban in 1960. Industrialization and urbanization of the country has been paralleled by a sizeable increase in the general standard of living of the majority of the population.

The economic structure, sensitively tied to the production and consumption of automobiles, has produced in this country the most mobile population in the world. Mobility, however, has tended to change not only the life of the individual American, but also the nature of the American city itself. Many medium-sized American cities have found themselves faced with problems which occurred formerly in only the largest of European cities.

The mobility of the middle classes has caused what has been described as "the flight to the suburbs," in order that the home owner can enjoy the less confined atmosphere of suburbia. The unskilled worker of rural America, finding his labor is no longer needed in a mobile and mechanized rural society has been drawn to the central city in search of employment. The overall pattern of the twentieth century American city is then a complex, highly developed, intensely productive central core area devoted to commerce, surrounded by the deteriorating "gray" areas housing the poorer classes of all races, and finally a rapidly expanding suburban area of lower density middle class housing. Traditionally in the last 40 years, the preferred mode of travel of the affluent American has been the automobile, with decreasing reliance on, and subsequent decreasing provision of public transit. This development has occurred on a widespread scale across the whole nation, bringing with it nationwide transportation problems, virtually identical from city to city, where highway officials struggle to provide facilities for a seemingly insatiable demand.

A cursory glance at a typical city might indicate to the uninformed observer that the city was planned in this manner. In most cases, however, the growth patterns are due to lack of planning, or occur in spite of planning due to the profit pressures exerted in a free enterprise society. In the past, too often, the effect of transportation facilities on developmental patterns has been underestimated and facilities have followed the demand of unplanned development.

The last 15 years of transportation planning have brought a deeper understanding of the nature of travel both in urban and rural areas. A realization has come about that travel is not a random phenomenum, but is predictably and closely related to the spatial separation of land uses (5). The older methods of designing transportation facilities based on past traffic trends have been supplanted by methods where consideration is given to the interaction of future land use and the future transportation network.

The predictability of travel trends has been demonstrated by the development of mathematical equations which relate travel patterns to zonal characteristics of a social and economic nature. These models are tested against existing patterns until their predictive ability is found adequately sensitive and accurate for satisfactory employment by the planner.

The model which determines how many trips originating in one zone will be destined to end in any other zone, is called the distribution model. Among the various types of distribution model in current use, the one with the greatest background of experience is the Gravity Model. One of its basic assumptions is that the probability of making a trip becomes less likely as trip length increases, and this relation of probability to distance is constant over the planning period for any particular city. The form of the relationship is currently expressed by an empirical curve known as the Travel Time Factor curve. The form of this curve has been the question of some debate in the last few years, both from the viewpoint of its exact nature and its constancy with respect to time.

The purposes of this thesis are therefore:

a. To determine whether the empirical travel curves can be replaced by mathematical functions of relatively simple form, amenable to numerical analysis.

b. To attempt to determine whether the travel time curves are random phenomena, peculiar to each city, or whether trends exist from city to city whereby certain characteristics of the curves may be associated with areal characteristics.

It is felt that where possible, all elements of an empirical nature should be removed from the planning process. If the travel time factor can be modelled, and the model is itself found predictable, one more step will have been made toward the deterministic description of human behavior.

CHAPTER II

THE DEVELOPMENT OF THE GRAVITY MODEL AND STATEMENT OF PROBLEM

The use of the theory of gravity to describe the interaction of human populations, dates back to the early nineteenth century. At that time H. C. Carey (24) stated that the theory of gravity influenced human interaction with direct ratio to the mass and inverse ratio to the distance; man was the unit of mass in society. In 1885, Ravenstein (25) restated this relation, in his "Laws of Migration," that the migration between two areas was a function both of the population at the source of migration, and the inverse of the distance between the source and the point of absorption of the migrant. This hypothesis was based on observations of migrant populations in North America, Europe and Africa.

That there was a gravitational effect relating to migration was confirmed by Young (26) in 1942. However, in his work, "The Movement of Farm Population," it was hypothesized that migration varied directly with the attractiveness of the destination, and inversely with the square of the distance. That distance entered into the gravity relationship in the form of an inverse square was confirmed by W. J. Rielly (27) in the 1920's. His work, relating to the retail trade areas of several medium-sized American towns concluded that:

> Under normal conditions, two cities draw retail trade from a smaller intermediate city or town in direct proportion to some

> power of the population of these two large cities, and in an inverse proportion to some power of the distance of each of the cities from the smaller intermediate city.

In another sociological gravity model, Dodd (28) indicated that the effect of distance on human interaction, was according to the inverse of distance, with unit exponent. Dodd, however, indicated that the relation might be more complex. Similar findings are reported by Cavanaugh (29).

Stewart (30,31,32) and Davis (33) indicated that the effect of distance in demographic change was an inverse relationship with unit exponent.

In 1946, Zipf (34) applied gravity theory to intercity movement of persons, finding that such movements could be related to the product of the populations of the cities divided by the distance between these cities. This was referred to as the "P_1P_2/D Hypothesis," which was applied to highway, rail and air traffic. In the 1920's, however, Pallin (7) investigated intercity movement by means of a gravity model and concluded that the exponent for distance should be 2.

Anderson (35) concluded that the exponent of distance was not constant in its effect on interaction, but was a variable inversely proportional to population size. This was one of the earliest works to propose a functional rather than constant form for the exponent of distance in gravity modelling.

Ikle, in work related to airline and automobile trips found remarkable variations of the exponent of distance in gravity model relationships (36). For urban automobile trips, an exponent of 0.69 was determined, while intercity trips had exponents ranging up to

2.57. He concluded, however, that exponents were constant with distance. It was suggested by Carrothers (37), also, that the impact of the distance exponent was not uniform, but varied with distance itself, rather than with population as suggested by Anderson.

In work relating to intercity telephone calls, Carroll (11) came to the conclusion that the effect of distance on intercity communication was of inverse exponential form. In a study relating to four Indiana cities, it was determined that the decay of interareal activity was dependent upon distance to an exponent 2.8.

With earlier work, the Michigan State Highway Department (14) had determined that the decay of trip making activity could be described by a model which included distance to a constant exponential power. This study noted that the value of the exponent varied by a trip purpose from 3.35 for short trips to 2.84 for trips to major regional centers.

The formalization of a gravity model for activity distribution fell to Casey (12) who restated Reilly's Law in a form similar to that used in present gravity model procedures. This formulation, which was a retail sales model, was expressed as:

$$B_{1.a} = \frac{\dfrac{Fa}{(D_{1.a})^2}}{\left(\dfrac{Fa}{(D_{1.a})^2} + \dfrac{Fb}{(D_{1.b})^2} + \cdots + \dfrac{}{(D_{1.z})^2}\right)} \quad B_1$$

Where B_1 was the buying power of neighborhood 1,

F_a, F_b, F_c, ... were the square feet of space in retail centers A, B, C ...

$D_{1.a}$, $D_{1.b}$, ... were the driving time distances between neighborhood 1 and the retail centers.

$B_{1.a}$ ... the purchases made in retail center A by residents of area 1.

Casey's work was limited to the formulation of a distribution model, and made no attempt to verify the authenticity of the constant exponent 2 used for the influence of distance.

Voorhees (5) utilized a similar formulation in his "General Theory of Traffic Movement." The model described in this work was a direct forerunner of the trip distribution model now widely used. This work used a constant exponent for the influence of distance, but recognized the need for a different exponent depending on trip purposes. It was suggested that for work trips the exponent should be 0.5, for shopping goods trips the value should be 2, and for all other trips the value should be 3. The gravity model as suggested by Voorhees was of the form:

$$T_{1.a} = \frac{\dfrac{S_1}{(D_{1a})^n}}{\dfrac{S_1}{(D_{1a})^n} + \dfrac{S_2}{(D_{2a})^n} + \cdots \dfrac{S_z}{(D_{2z})^n}} T_A$$

Where T_A = Total trips produced for A for the trip purpose.

$T_{1.a}$ = Trips for this purpose from 1 to a.

S_i = Measure of attraction of ith zone for trips of this purpose.

D_{ia} = Distance from A to ith zone.

n = Some undetermined exponent, values of which were recommended.

This form of model was used in the Hartford Study (23). Current gravity model practice (4,7) utilizes a similar formulation with the exception that the distribution model uses the number of trips attracted as the measure of attraction. It is now recommended that the calculation of zonal attractions be carried out in the trip generation phase of the planning process. Current practice also recognizes that the exponent of distance may not be constant. Use is made of a travel time factor, which is equivalent to the use of an exponent varying with distance. Voorhees (16,5) also indicated that travel time rather than physical spatial separation was a realistic measure of impedance to travel. This is reflected in current practice which uses the following form of gravity model (4,7):

$$T_{ij} = P_i \frac{F_{ij} \cdot A_j}{\sum\limits_{\text{all } k} F_{ik} A_k}$$

Where T_{ij} is the trip interchange from i to j.

P_i is the number of trips produced at zone i destined to all zones.

A_j is the number of trips attracted to zone j from all zones.

F_{ij} is the friction factor derived from the travel time curve for a travel time equalling that time from i to j.

For clarity of presentation the model is shown without the social-economic adjustment factors which are often found necessary.

In early work in the Washington area, Hansen (6) used travel time as a measure of spatial separation, and found that a constant exponent was not usable. This exponent appeared to increase with increasing separation. This was highly apparent in the case of the work trip. The need for a variable exponent in the use of a gravity model has been found necessary in many city studies carried out since Hansen's work in Washington. The San Mateo Study, for example, used travel time exponents which varied with time, ranging from 0 to 1.2 (21). More common practice is the use of ordinates from an empirical hand drawn "friction factor curve." equivalent to using an exponent of time which varies the whole time range.

Tanner investigated the question of a constant exponential of spatial separation from a mathematical approach (17). It was found that it was not mathematically possible for the distance exponent to remain constant with the distance. This work indicated that short trips required, under assumptions of uniform population density, an exponent between 2 and 3, which was theoretically impossible in the use of long trips, leading to a ridiculously high vehicle mileage of travel in an area. Rather than a constant exponent where:

$$f(d) = d^n$$

Where $f(d)$ is the functional form of the effect of distance as it would appear in a gravity model formulation.

d is the spatial separation.

n is some constant.

Tanner suggested a more general form of curve, the Gamma function:

$$f(d) = e^{-\lambda d} d^{n}$$

Where $f(d)$ is the functional form of the effect of distance as it would appear in a gravity model formulation.

λ is some non-negative constant.

d is the measure of spatial separation.

n is some constant.

e is the base of natural logarithms.

Such a form would permit a sufficiently rapid decay of the function to prevent errors in long trip computations.

The constancy of the travel time factor over the period of planning has been the subject of wide discussion. Based on experience in Baltimore (16), and Washington (18), this assumption would appear to be valid. Extensive testing of this assumption has been impossible because of inadequate data taken in old origin-destination studies, but will become more feasible when the studies made in the late 1950's can be used as base data.

An examination of a range of travel time factors of various cities was made by Whitmore (1). This study found that the travel time factor could be represented by general polynomials, and that regional friction factors are similar from region to region, but vary considerably from city to city. Whitmore indicated that the best fit to travel time factors could be found with a polynomial of the form:

$$f(t) = A_0 \quad A_1 t \quad A_2 \left(\frac{1}{t - 3}\right)$$

This function gave acceptable fits over medial ranges only of travel time. An examination of the form of function indicates that as t tends to infinity, the function itself becomes infinite. While it gives apparently adequate fit, over certain ranges, it cannot be held as a completely rational form of the travel time factor. Possible forms should certainly have characteristics similar to that of the function suggested in Tanner's work, where the value of the function decreases at an increasing rate with time. A further limitation on the findings of the Whitmore study comes from the treatment of data where both overall travel times and driving times were used. To enable the use of these two different types of times, driving time curves were converted to travel time curves by the addition of four minutes to all driving times. The validity of this treatment is questionable, since terminal time is neither constant throughout a city, nor constant from city to city.

It has already been stated that one of the tacit assumptions of the Gravity Model as currently employed in transportation planning, is that the calibrated model is constant over the planning period. The form of the model in current use is (4):

$$T_{ij} = \frac{P_i \; A_j \; F_{ij} \; K_{ij}}{\sum\limits_{\text{all } j} A_j \; F_{ij} \; K_{ij}}$$

Where T_{ij}, P_i, A_j, F_{ij} are as previously defined.

K_{ij} is the zone-to-zone adjustment factor.

The input requirements for the right-hand side of this model are either known or assumed. The zonal attractions and productions for the design year are estimated from land use models, and the zone-to-zone adjustment factors are either considered constant with time (4), or are calculated from land use considerations on a zone-to-zone basis (2,18). It is assumed, however, that the travel time factors do remain constant throughout the planning period.

This assumption of constancy has been questioned from time to time (1,16). Part of the difficulty is determining whether in fact such an assumption is justified, is due to the empirical nature of the travel time curves when calculated in accordance with the Bureau of Public Roads recommendations. It has been recognized for many years that Voorhees' original conception of a gravity type effect, with the influence of distance being related to some constant power (5), has not generally been observed in most cities (6) Current practice therefore reflects these observations by hand fitting an empirical curve to travel time factors (4,7). The use of an empirical or form-free curve has inherent problems. The assumption of constancy of travel time factor can be tested only by comparing the predicted interareal travel patterns with actual travel patterns observed at the end of the planning period. In the case of many cities with completed transportation studies, the end of the planning period is still many years away, and such a test is not yet possible. Even an evaluation on the basis of interim travel patterns is still not yet possible, since the bulk of transportation studies are under seven years old at this date. Where such a test has been possible, the results have indicated simply that the travel time factors could have remained constant over the planning period. A study was run for the Washington, D.C. highway complex to determine whether the 1955 origin destination patterns could be projected from the 1948 origin-destination study and 1955 travel time factors. Because of the comparison of a before and after situation, formalized statistical testing was not possible; the conclusions therefore were that the travel time factors could have been assumed constant over the planning period without large error.

It is felt that a great deal of information on the behavior of the travel time factor is being lost by current use of empirical travel time factors. This problem has been recognized for some time by the Bureau of Public Roads which states (4):

> It is important to keep the "line of best fit" smooth and as straight as possible for the following reasons:
>
> a. Smooth curves can be approximately defined in a mathematical expression; possibly, one that is not complex.
> b. If these curves can be approximated by a mathematical expression, meaningful comparisons can be made between these expressions for different urban areas with various population and density characteristics.
> c. These comparisons would eventually help quantify, with a mathematical function, the effect of spatial separation between zones on trip interchange.

If nonparametric curves continue to be used in the gravity models of transportation studies, little advance can be made with respect to the assumption of constancy. This assumption is not likely to be greatly in error in a slowly developing community. In an area of considerable development, and significant social change, the assumption may well be unjustified. It is precisely within areas of radical change, that the transportation modelling process is of greatest value.

The use of parametric travel time curves is recommended in order that information relating the form of the curve to the character of the study area can be retrieved by statistical relationships. Statistical modelling is in widespread use throughout the remainder of the transportation planning process, and it is felt that it can well be extended into the modelling of the travel time factor itself. Any model which is used to describe the form of the travel time factor must be sufficiently flexible to fit the various shapes that the factors have been found to have. The over-simplifying assumption of distance to some uniform power (5), was earlier found inadequate (6). Whitmore experimented with a polynomial type of fit to data similar to that used in this work. The fit achieved from this approach was adequate but was limited to medium lengths of travel time.

It is therefore felt that a more flexible curve system is necessary to achieve good fit to the type of curve encountered with the travel time factor. The Pearson System of curves, a highly flexible system derivable from one basic differential equation appears to have adequate variety of form to permit its adaptation for use as a travel time factor curve. Pearson curves have as many as three shape parameters and one shift parameter. All curves are derivable from the basic formula:

$$f(x) = \frac{x - a}{b_1 + b_2x + b_3x^2}$$

where a, b_1, b_2, b_3 are constants.

It was decided that this research would determine how satisfactorily this curve system could serve as travel time factors.

A second stage examination involved the evaluation of the existence of statistical relationships between the characteristics of the study area and the curve parameters.

The second stage examination leads to a minor purpose of the research. If the parameters of the Pearson curves could be modelled by regression techniques, a rational first estimate of the travel time curve could be obtained, which could then be modified according to the recommended Bureau of Public Roads procedure. Current methods of estimating the travel time factor curve for the first calibration cycle generally evolve on a guess based on past experience in other study areas. An improvement on this technique would come about from a modelling approach. Such an approach would decrease the number of cycles needed to calibrate the gravity model within required limits of accuracy, with an ensuing decrease in computer and technical time involved in the calibration process.

Further advantages from the use of parametric curves occur from the fact that the calibration cycle itself can be completely computerized, eliminating the present methods of hand-fitting empirical curves at each stage of calibration of the gravity model. The modelling method indicated in this work is directly applicable to the fitting of a parametric curve to points, rather than the hand-fitting method recommended by the Bureau of Public Roads (4,7). The current procedure involves the calculation of a new factor, based on the relation of the observed trip length frequency distribution, and the distribution computed from the last cycle of the gravity model. The travel time factor used in the next cycle of the gravity model is a hand-fitted curve to these calculated points. Utilizing the fitting procedure outlined in this work, the gravity model can be made to converge completely within the computer with subsequent reduction in requisite time.

CHAPTER III

PROCEDURE FOR INVESTIGATION

In order to achieve valid statistical information from the parameters of mathematical curves which had been fitted to the travel time factors, it was essential that a very high degree of fit should be obtained between the model and the travel time factor. It was also imperative that the fitting procedure should reflect the trip length frequency distribution, in that those time intervals with best fit to the shape of the curve should be those intervals containing the major portion of trips.

Several methods of curve fitting were attempted before any satisfactory fit was achieved. Fitting by the method of finite differences for a nonlinear least squares fit proved quite inadequate. This method was attempted with a variety of curve forms including:

Double Exponential

$$F(t) = C_1 \, e^{at^b} \, e^{-ct}$$

Shifted Double Exponential

$$F(t) = C_1 \, e^{a(t-d)^b} \, e^{-c(t-d)}$$

Pearson VIII

$$F(t) = b\left(1 + \frac{t - c}{a}\right)^{-d}$$

Pearson I

$$F(t) = \frac{C_1}{a^{b+c+1}} \cdot \frac{\Gamma(b + c + 2)}{\Gamma(b + 1)\ \Gamma(c + 1)} \cdot (t - d)^b\ (a - t + d)^c$$

Pearson III

$$F(t) = C_1\ \frac{a}{b} \cdot \frac{(a + 1)^b}{e^{a+1}\ \Gamma(a + 1)} \cdot \left(1 + \frac{t - c}{b}\right)^a \cdot e^{-\frac{b}{a} \cdot (t-c)}$$

Where a, b, c, d, C_1 are constants.

e is the base of natural logarithms.

$\Gamma(x) = \int_0^\infty e^{-z}\ z^{x-1}\ dz$, (the gamma integral).

t is the time separation.

$F(t)$ is the functional form.

It is sufficient here to summarize the findings that the method of finite differences was found inadequate using either the natural curve form, or transposing into logarithms and fitting the log transformed curve. Fitting the natural curve resulted in good fits at the lower range of travel time, but extremely poor estimates of the true curve at highest travel times. By transforming the curves into logarithmic form the poor estimates at larger travel times were considerably improved, but the curve fit achieved at short travel times was poor, and it was at these shorter travel times that the greatest number of trips occurred.

Another method of fitting attempted was a nonlinear least squares fit of the travel time curve multiplied by a weighting function $e^{\alpha t}$. This weighting function greatly improved the fit. It was necessary, however, to choose subjectively the value of α, which was found to be sensitive for the fitting procedure, and no single α value could be used for all curves.

With these findings, nonlinear least squares fitting was abandoned since it was felt that a minimization of the residuals would not achieve an adequate fit for the exponential shape of curve involved in the travel time factor curve.

Curve fitting was therefore carried out by the method of moments, which was found to give excellent results, with Pearson I and Pearson III distributions. For a full coverage of the method of moments as applicable to the Pearsonian system of distributions, the reader is referred to Elderton's work (3). Appendices I, II and III contain the mathematical derivations of the curves and curve criteria used for fitting these distributions to observed curves.

The Pearson system of curves may be derived from the basic differential equation:

$$\frac{dy}{dt} = \frac{y(t + a)}{b_0 + b_1 t + b_2 t^2 + \cdots}$$

Where y is F(t), the functional form.

t is the time separation.

a, b_0, b_1, b_2 are constants.

In practice the series in the denominator is limited to three terms, giving a curve with a maximum of three shape parameters. Pearson (22) has pointed out that the gain in introducing additional constants beyond four, leads to a great deal of additional work with little benefit. The higher moments required to determine these constants when performing curve fits can become unreliable. The basic differential equation is therefore of the form:

$$\frac{dy}{dt} = \frac{y(t + a)}{b_0 + b_1 t + b_2 t^2}$$

This is a very general form of curve, which by proper choice of constant can ensure that y = 0 as t tends to infinity, and that dy/dt = 0 when y = 0. The equation therefore conforms with observed properties of the travel time factor, which Whitmore's function fails to do.

Depending on the values of the constants in the basic differential equation, a variety of curve types are obtainable. If the roots of the denominator are real and of different sign then the differential equation gives the Pearson I curve which is of the form:

$$F(t) = \frac{C_1}{A^{m_1+m_2+1}} B(m_1, m_2)\ (t - c)^{m_1}\ (A - (t - c)^{m_2})$$

Where m_1, m_2, A are shape parameters.

c is a shift parameter.

C_1 is a constant.

t is the time separation.

$B(m_1, m_2)$ is the value of the Beta function with parameters m_1, m_2.

In this case $k < 0$. (See Appendix I for definition of k, the curve criterion, which is equal to $b_1^{\,2}/4\, b_0\, b_2$). When $b_2 = 0$, the criterion $k = \infty$ and the curve solution to the differential equation is:

$$F(t) = C \cdot \frac{p}{A} \frac{(p + 1)^p}{e^{p+1}\ (p + 1)} (1 + \frac{t - \mu}{A}) \cdot e^{-p/A(t-\mu)}$$

Where p, A are the shape parameters.

μ is a shift parameter.

C is a constant.

t is the time separation.

F(t) is the functional form.

The value of k can be estimated from the first four moments of the curve about its mean value. Depending on the value of k, β_1, β_2 (see Appendix I), 11 different types of curve result. These are the Pearsonian System of curves. For the purposes of this work, it was found that satisfactory fit could be obtained using Pearson I for work trips and non home based trips where k was found to have a low negative value, and Pearson III for shopping trips where k was found to have a high value and $2\beta_2 \simeq 6 + 3\beta_1$.

It is of considerable interest at this point to compare this approach from the differential equation basis with Tanner's earlier observation that the travel time factor could not be constant, but was probably of the form:

$$f(d) = e^{-\lambda d} d^n .$$

In Appendix I, equation 4 for the Pearson III distribution is

$$y = y' e^{x/b_1} (b_1 x + b_0)^{(a-b_0/b_1)/b_1}$$

If $b_0 = 0$, then

$$y = y' e^{\frac{1}{b_1} \cdot x} \cdot (b_1 x)^{a/b_1}$$

which is of the form:

$$y = k e^{mx} x^n$$

Tanner's suggested form of the travel time curve which was derived mathematically and was unsupported by experimentation, is a special case of the Pearson III distribution, itself a special case of the basic differential equation. The work undertaken in this thesis can therefore be considered an examination of Tanner's generalized hypothesis.

The general method of curve fitting to fit a parametric model to the travel time factors was basically similar for all trip purposes, the decision of which Pearson curve to use being based upon the value of the criterion k.

No detailed discussion of the method of moments is included here. The reader is referred to Appendices I, II and III and Figure 44, for an outline of the procedure utilized in obtaining an iterative curve fit.

One departure from the curve fitting procedure indicated in Appendices II and III was the use of a weighting factor to weight the value of the lack of fit value, D_t.

The lack of fit of the model with respect to the travel time curve was expressed in relationship somewhat similar to the Chi-square form:

$$D_t = \frac{(\text{Model Value}_t - \text{Travel Time Curve Value}_t)^2}{\text{Travel Time Curve Value}_t}$$

$$= \frac{(\text{Error}_t)^2}{\text{Travel Time Curve Value}_t}$$

If the percentage of trips occurring at time t = W_t, then $\sum_{\text{all } t} D_t W_t$ is an estimate of the degree of fit, weighted by the trip length distribution. Improved curve fitting over the first fit by moments was obtained by a cyclic computer program which re-estimated a curve fit from the fit on the last cycle, and the residual error. It was found that a local minimum of $\sum D_t W_t$ occurred, and this was taken as the best fit of the travel time curve by moments. The method is outlined in Figure 44 and Appendix IV.

The degree of fit attained by the above procedure was measured by two criteria, both related to the variance of the one minute ordinates on the travel time curve.

The index of multiple correlation (ρ) is a measure of the amount of variation explained by the model with respect to the total variation. It is more meaningful than the coefficient of correlation (r) when used for non-linear models, since the coefficient of correlation is based on linear relationships.

If the S^2 is the total variation of the model ordinates taken at one minute intervals, and S_e^2 is the variation of the residuals at the same intervals after modelling, then

$$\text{Index of Multiple Correlation} = \rho^2 = 1 - \frac{S_e^2}{S^2}$$

The limits of ρ are 0 and 1. A value of $\rho = 0$ would indicate that the curve form cannot be expressed by the model equation, while $\rho = 1$ would indicate perfect coincidence of the model and the travel time curve.

The F-ratio is the ratio of the variation explained by the model to the residual variation.

$$\text{F-ratio of regression} = \frac{S^2 - S_e^2}{S_e^2}$$

The limits are zero and infinity with high F-ratios being indicative of closely correlating models.

The relationship between the parameters of the Pearson I and Pearson III models at best fit, and the city wide variables was examined chiefly by means of correlation analysis, involving the calculations of the coefficient of correlation (r).

The coefficient of correlation is a measure of the degree of change in one variable associated, on the average, with a given change in the other. The correlation can be either positive or negative. When positive, one variable tends to increase as the other increases; when negative an increase in one variable tends to give a decrease in the other. The value of the correlation coefficient lies between -1 and 1. A value of 0 indicates no association between the two variables, while an absolute value of 1 indicates a perfect linear relationship.

The correlation coefficient may be computed from:

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

Where r is the correlation coefficient.

n is the number of variates.

x_i is the ith dependent variate.

$\bar{x}$ is the mean of the x variates.

y_i is the ith independent variate.

$\bar{y}$ is the mean of the y variates.

Based on close associations between the model parameters and the city-wide variables, regression equations were obtained to formalize the relationship between these dependent and independent variables. In some cases, where an examination of the data indicated that a simple linear regression equation could not totally describe the relationship, nonlinear equations were formulated by linear regression on the log transformed variables.

Where a better relationship could be found between a curve parameter and more than a single city variable, a multiple linear regression equation was determined, with a joint regression of the dependent curve variable on two independent city variables. The ability of the regression equation to relate independent and dependent variables was measured with several statistical quantities, including the standard error of estimates, the multiple correlation coefficient, the F-ratio of the regression line, the partial correlation coefficients and levels of significance of the regression coefficients. For an explanation of these statistical measures, reference is made to statistical texts where extensive coverage is possible (9,10).

CHAPTER IV

DISCUSSION OF RESULTS OF MATHEMATICAL MODELLING

Since the purpose of this research was the determination of the existence of statistical trends within travel time factors subdivided by purpose for cities of widely-diverging character, it was essential that the source data of travel time factors should be homogeneous in character. Ten cities were selected, each having "auto-driver driving-time" travel time factors. Considerable differences were found within the cities in the manner in which total travel was subdivided by purpose. Among the different purposes found were work, school, major shopping, convenience shopping, social-recreational, miscellaneous, non home based and home based other than work. Only three purposes were used for this study; these were the work, shopping and non home based purposes. The other purposes could provide only very small sample sizes which would have made the determination of statistical trends of dubious value.

Homogeneous data were available for ten cities for the work trip, nine cities for the non home based trip, and five cities for the shopping trip. The reason for the small size of the shopping trip sample was the various ways in which the trip was classified in different studies. In many cases, the shopping trip was further subdivided by purpose into convenience shopping, and major apparel and furniture shopping. Other studies combined personal business and shopping into a commercial trip classification. In the smaller studies, the subdivision into purpose was at a much lower degree of detail than in the larger studies. Typically in such a study the division of total trips would be into the purposes of home based work, non home based trips and home based other than work trips. In such a study, shopping trips would be contained in the last of these classifications.

For clarity of reference, the sections dealing with work, non home based and shopping trips have been written without cross reference. Some of the findings and discussion contained in one section are repeated in the other sections where applicable. It was felt that the repetition involved was outweighed by the ability of a reader interested only, for example, in non home based trips to extract all the pertinent findings from a perusal of the non home based section only.

Work Trips

It was found that the travel time factor for work trips could be modelled satisfactorily by the use of the Pearson Type I curve, which has three shape parameters and one shift parameter. The summary of the results is contained in Table 1 and Table 2. The Pearson I curve was found to model adequately a full range of travel times which included at least 90 per cent of all travel for that purpose. The range of times varied from a low value of two minutes to a high value of 50 minutes. In all cases the percentage of trips falling outside the upper limit of the model was sufficiently small that it could be ignored without affecting the validity of the model.

Table 1. Summary of Results--Work Trip Travel Time Factor for Ten Cities

Cedar Rapids	$F(t)_1 = \dfrac{N_1}{55.9^{2.21}} \cdot \dfrac{\Gamma(3.21)}{\Gamma(0.73) \cdot \Gamma(2.48)} (t - 0.72)^{-0.27}(56.6 - t)^{1.48}$
Waterbury	$F(t)_2 = \dfrac{N_2}{74.2^{4.44}} \cdot \dfrac{\Gamma(5.44)}{\Gamma(0.27) \cdot \Gamma(5.16)} (t - 1.71)^{-0.73}(75.9 - t)^{4.16}$
Erie	$F(t)_3 = \dfrac{N_3}{40.9^{2.52}} \cdot \dfrac{\Gamma(3.52)}{\Gamma(0.62) \cdot \Gamma(2.89)} (t - 1.21)^{-0.38}(52.4 - t)^{1.89}$
New Orleans	$F(t)_4 = \dfrac{N_4}{70.7^{3.12}} \cdot \dfrac{\Gamma(4.12)}{\Gamma(0.27) \cdot \Gamma(3.85)} (t - 1.93)^{-0.73}(72.0 - t)^{2.84}$
Providence	$F(t)_5 = \dfrac{N_5}{104.6^{5.74}} \cdot \dfrac{\Gamma(6.74)}{\Gamma(0.27) \cdot \Gamma(3.85)} \cdot (t - 2.25)^{-0.66}(106.9 - t)^{5.40}$
Sioux Falls	$F(t)_6 = \dfrac{N_6}{15.8^{1.13}} \cdot \dfrac{\Gamma(2.13)}{\Gamma(0.65) \cdot \Gamma(1.48)} \cdot (t - 0.97)^{-0.35}(16.8 - t)^{0.48}$
Hartford	$F(t)_7 = \dfrac{N_7}{60.6^{3.38}} \cdot \dfrac{\Gamma(4.38)}{\Gamma(0.27) \cdot \Gamma(4.01)} (t - 1.10)^{-0.63}(61.7 - t)^{3.01}$
Fort Worth	$F(t)_8 = \dfrac{N_8}{54.8^{5.01}} \cdot \dfrac{\Gamma(6.01)}{\Gamma(0.12) \cdot \Gamma(4.89)} \cdot (t - 2.23)^{-0.88}(64.6 - t)^{3.89}$
Baltimore	$F(t)_9 = \dfrac{N_9}{57.3^{2.54}} \cdot \dfrac{\Gamma(3.54)}{\Gamma(0.35) \cdot \Gamma(3.19)} \cdot (t - 1.87)^{-0.65}(59.2 - t)^{2.19}$
Los Angeles	$F(t)_{10} = \dfrac{N_{10}}{128.3^{7.10}} \cdot \dfrac{\Gamma(8.10)}{\Gamma(0.23) \cdot \Gamma(7.87)} \cdot (t - 11.37)^{-0.77}(126.3 - t)^{6.87}$

General curve form is Pearson I:

$$F(t)_i = \frac{N_i}{A^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot (t-c)^{m_1} \cdot (A - (t-c))^{m_2}$$

where m_1, m_2, and A are shape parameters and c is a shift parameter. N_i is a constant which modifies the magnitude of the curve ordinates, not affecting the shape or function of the travel time factors.

Table 2. Summary of Pearson I Shape Parameters for Home Based Work Trips

	m_1	m_2	A	c Shift Parameter
Cedar Rapids	-0.27	1.48	55.9	0.72
Waterbury	-0.72	4.16	74.2	1.71
Erie	-0.37	1.89	40.9	1.21
New Orleans	-0.73	2.84	70.7	1.93
Providence	-0.66	5.40	104.6	2.25
Sioux Falls	-0.35	0.48	15.8	0.97
Hartford	-0.63	3.01	60.6	1.10
Fort Worth	-0.83	3.89	54.8	2.23
Baltimore	-0.65	2.19	57.3	1.87
Los Angeles	-0.77	6.87	128.3	11.37

No attempt was made to fit at very low travel times for two reasons. Firstly, the number of work trips less than four minutes constitutes only a small percentage of all work trips. Secondly, numerous observers in the field of transportation planning have expressed the opinion that travel times of less than five minutes are meaningless because of inaccuracy of estimate by the trip maker at the time of the origin-destination study, and a measure of under reporting of the number of trips made where the length of trip is small (1).

Table 7, Appendix A, shows a comparison of the Pearson Type I model with the actual travel time factor used in the city or regional transportation study. Table 7 also shows the index of multiple correlation of the actual curve and the model. Also shown is the F-ratio of the variance of the unmodelled travel time values to the variance of the residuals about the model. It is apparent that a high degree of fit has been obtained by the use of the Pearson I curve. Perfect correlation between the model and the travel time factors would give an index of multiple correlation of 1.0 and an infinite F-ratio. The table shows that very high F-ratios were obtained, ranging from 26,733 to 831; high relative values of the index of multiple correlation were also obtained. The latter ranged from 0.999 to 0.979. Figures 20 through 29, Appendix A, show graphically the relation between actual travel time curves and the Pearson I models.

Relationships between Curve Parameters and Area Characteristics

The second stage of the research dealt with attempting to find statistical relationships between the parameters of the model for travel time factors, and various city-wide variables. Such a relationship would indicate predictability of the travel time factor curve under varying conditions, and would shed light on the assumption that travel time factors are constant with time. This is a basic assumption in the calibration procedure for the gravity model which has been questioned (1,2).

Table 22, Appendix D, shows that statistically significant trends were found to exist between the shape parameters of the model and city-wide variables determined in the origin-destination studies. The selection of the variables used in the regression was based both on the correlation analysis, and the suitability of the variable for predictive purposes. In general those variables with the highest correlation coefficients were used to form regression equations. Where it was possible to avoid variables involving the study area size, this was done. For predictive purposes, such variables would in general be unreliable, since the inclusion of large peripheral rural areas could radically affect the value of such variables without change on the trip characteristics. Final selection of the regression equation was also predicated on minimizing the significance level both of the regression coefficients of the independent variables, and the regression equation itself. This procedure was followed for all trip purposes.

Table 8, Appendix D, and Figure 1 indicate that the parameter m_1 could be modelled by:

$$m_1 = -0.993 + 0.000933 \begin{pmatrix} \text{Home based work trips per} \\ \text{1,000 population} \end{pmatrix}$$

The regression equation was significant at the 2 per cent level. From the regression analysis there is strong indication that as trip making intensity increases for the work purpose, there is a corresponding increase in the value that can be expected in the parameter m_1.

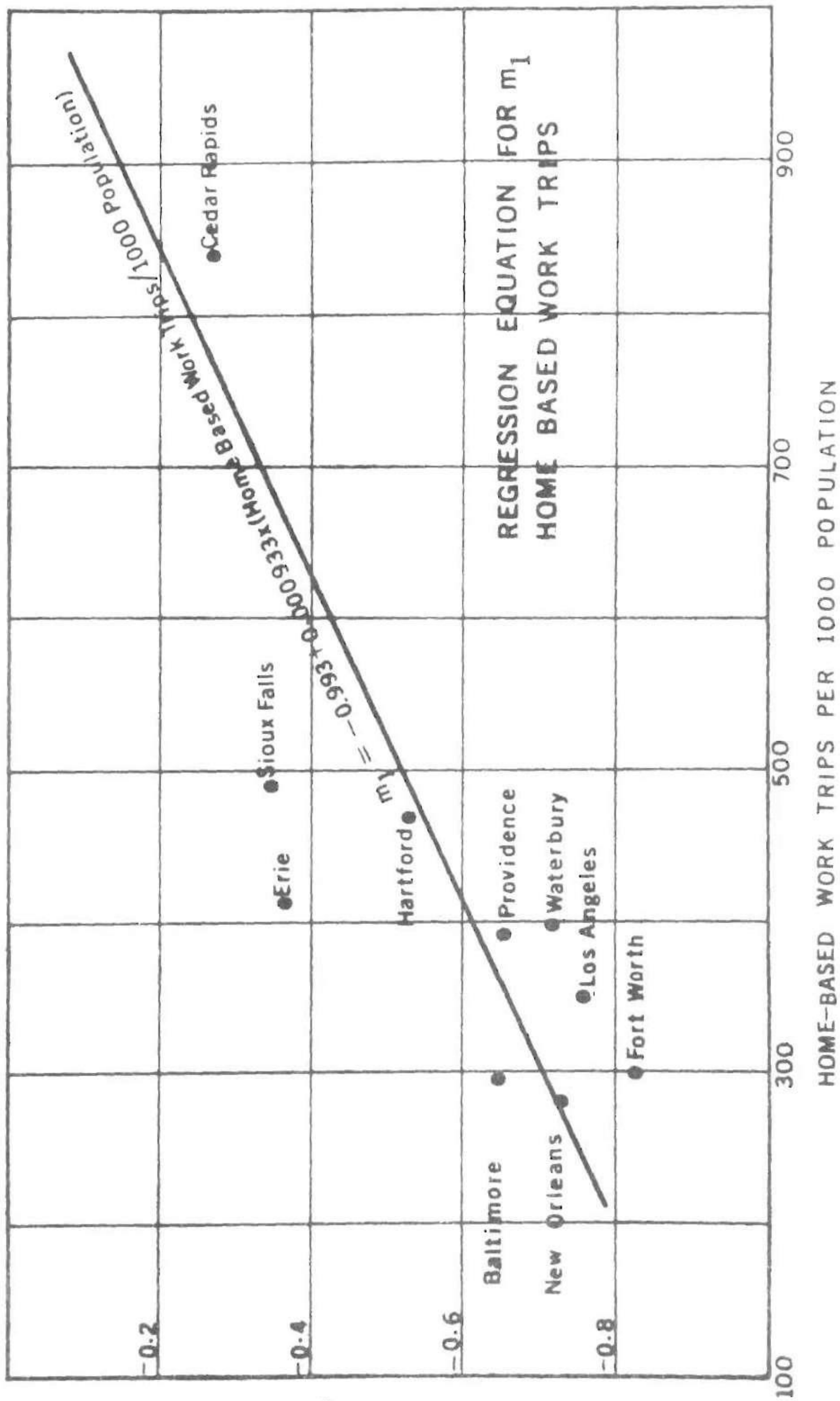


Figure 1. Regression Equation for m_1, Home Based Work Trips

Table 9, Appendix A, and Figure 2 show that the shape parameter m_2 for home based work trips was found to correlate with the total trips per car within the study areas. It was found that the most satisfactory regression equation was of the form:

$$\ln (m_2) = 3.51 - 1.74 \times \ln (\text{total trips per car})$$

This equation was significant at the 1 per cent level, indicating a strong statistical probability that the value of m_2 decreases with the total trip making intensity per car occurring within the study area. Such a correlation would indicate that as trips per car decrease, the slope of the travel time curve decreases at lower travel times, see Figure 2.

A very strong relation between the third shape parameter and the total number of home based work trips was found to exist. This third shape parameter is a measure of the spread between the upper and lower bounds of the Pearson I distribution. Table 10, Appendix A, and Figure 3 indicate the descriptive regression equation which was found to have the following form:

$$\ln A = -4.955 \times 10^4 \times (\text{Total Home Based Work Trips})^{-1} + 4.52$$

or

$$A = 91.83 \times e^{\frac{-4.955 \times 10^4}{(\text{Total Home Based Work Trips})}}$$

This equation was significant at the 1 per cent level.

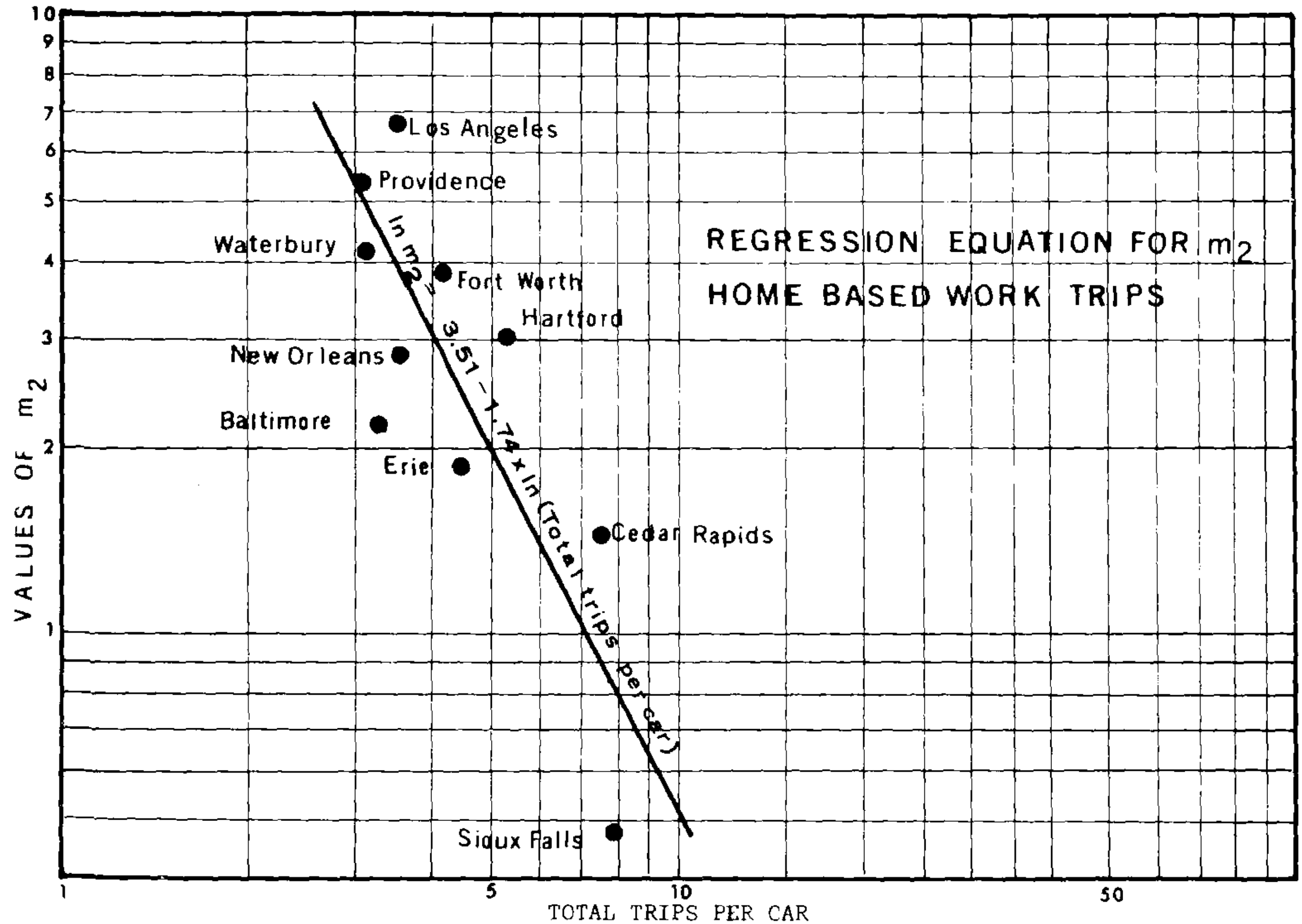


Figure 2. Regression Equation for m_2 , Home Based Work Trips.

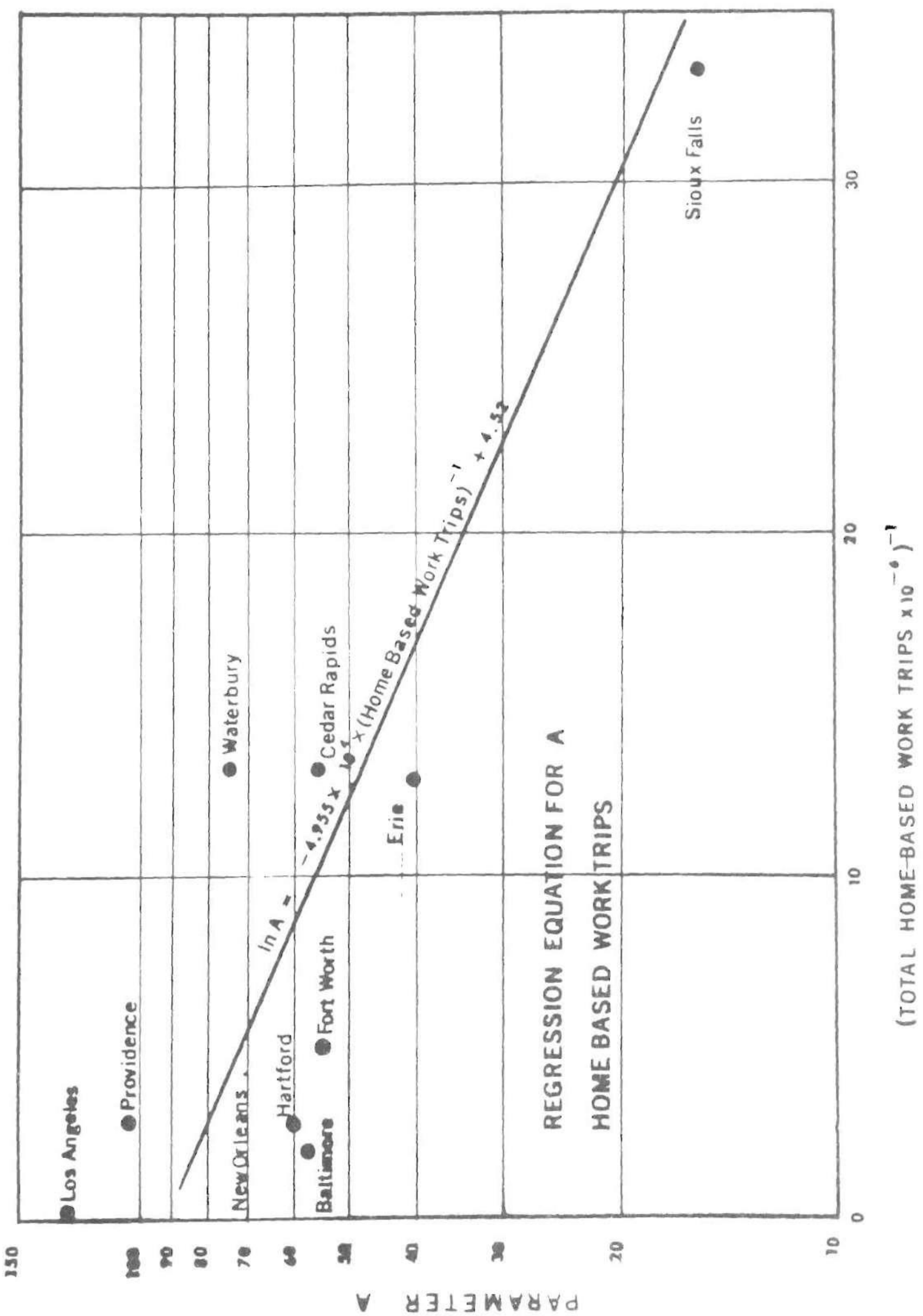


Figure 3. Regression Equation for A, Home Based Work Trips

As stated, there was a strong tendency for A to increase with an increase in the total number of home based work trips in the area. The relationship was not found to be linear, but rather of exponential form, with A increasing more slowly as the home based work trips in the area increased. The regression equation indicates that a limiting value of A exists at $e^{4.52}$ or 91.8. This presence of an upper limit, a consequence of using the Pearson I model, is rational in that while travel has been seen to decay at a rate close to the exponential rate, it is unreasonable to anticipate travel for urban purposes to continue to extremely long travel times. It appears that trip makers set a limit upon the time to be spent for certain trip purposes. As the urbanized area increases, the number of home based work trips increases, and the traveller appears to be prepared to spend a greater amount of time. Long travel times are not found in small urbanized areas.

The fourth and last parameter of the Pearson I model is c, which can be regarded as a shift parameter. In all cases of the models by moments, c was found to be positive, and ranged from 0.72 to 2.25 for compatible data. It was found that c could be correlated with home based trip productions per 1,000 population, r = 0.77. Table 11, Appendix D, and Figure 4 indicate the linear regression equation obtained from this variable:

$$c = 2.63 - 0.0025 \times \begin{Bmatrix} \text{Home Based Work Trips per} \\ \text{Thousand Population} \end{Bmatrix}$$

This equation was found to be significant at the 2 per cent level.

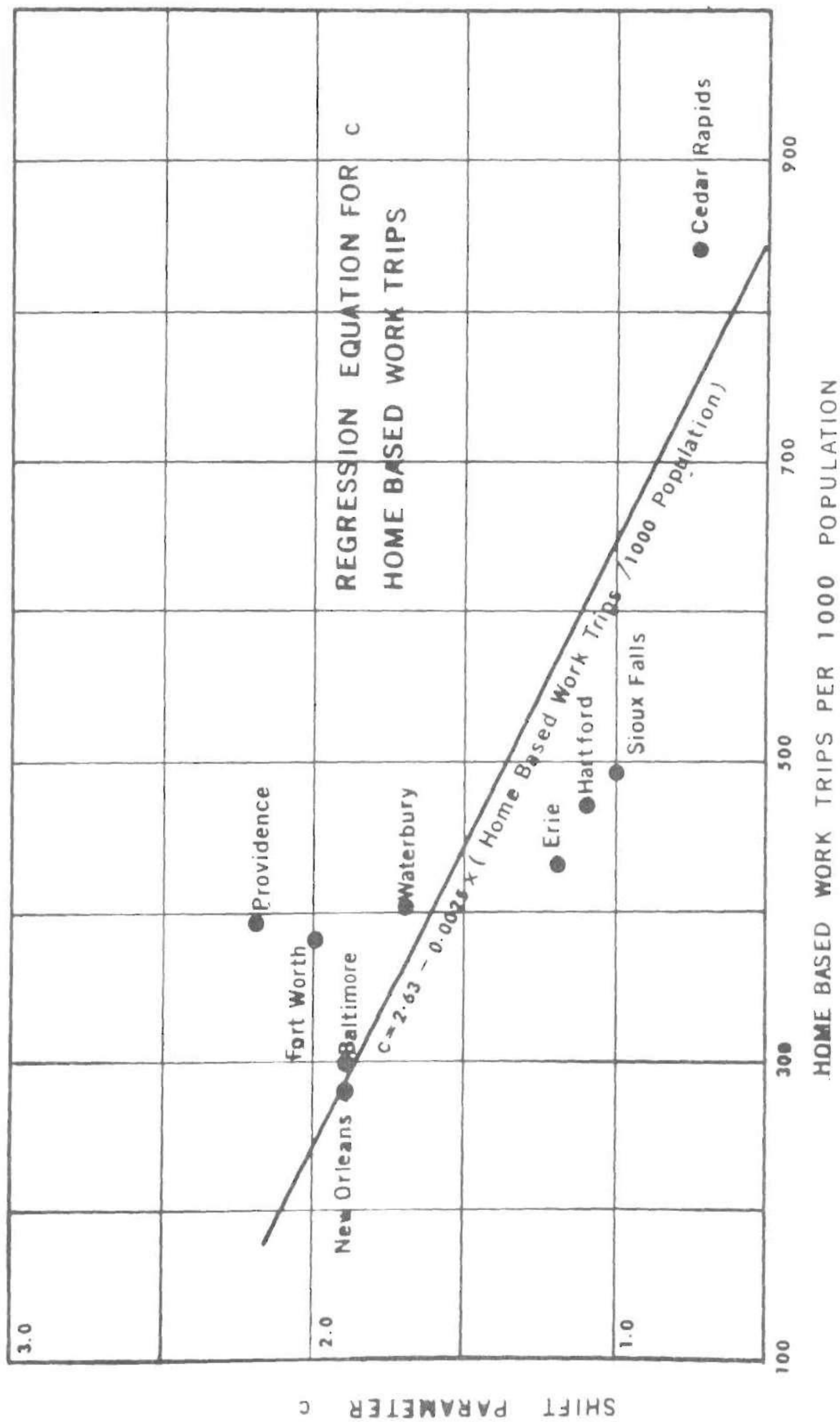


Figure 4. Regression Equation for c, Home Based Work Trips

Although the standard error of the equation was high compared to the range of the observed values, the predicted c values were close in absolute value to observed c values. It is apparent that as home based work trip intensity increases, the value of c decreases, with a reasonable statistical correlation.

Sensitivity Analysis

An examination of the sensitivity of the parameter m_1 is displayed in Figure 5. The effect of both increases and decreases of 10 and 50 per cent of the determined value of m_1 is shown for a home base work model. It is apparent that the parameter m_1 is sufficiently insensitive to present no problem in the use of the Pearson I model. A 10 per cent change in the value of the parameter affects the model only slightly. An increase in m_1 produces a greater negative slope at lower travel times and a decreased slope in the range of longer travel times.

Figure 6 shows the sensitivity of the parameter m_2. A 10 per cent change in the value of the parameter does not greatly alter the general shape of the curve. A 10 per cent increase in the parameter produces a small increase in the values of the travel time curve at the lower time range and insignificant change over the middle ranges of time. Greatest change to the curve ordinates are produced at the largest travel times. Figure 6 also indicates the effect of 50 per cent change in the value of m_2.

The parameter m_2 like the parameter m_1 is only slightly sensitive, i.e. small changes in either parameter do not bring about large shape changes in the Pearson I curve.

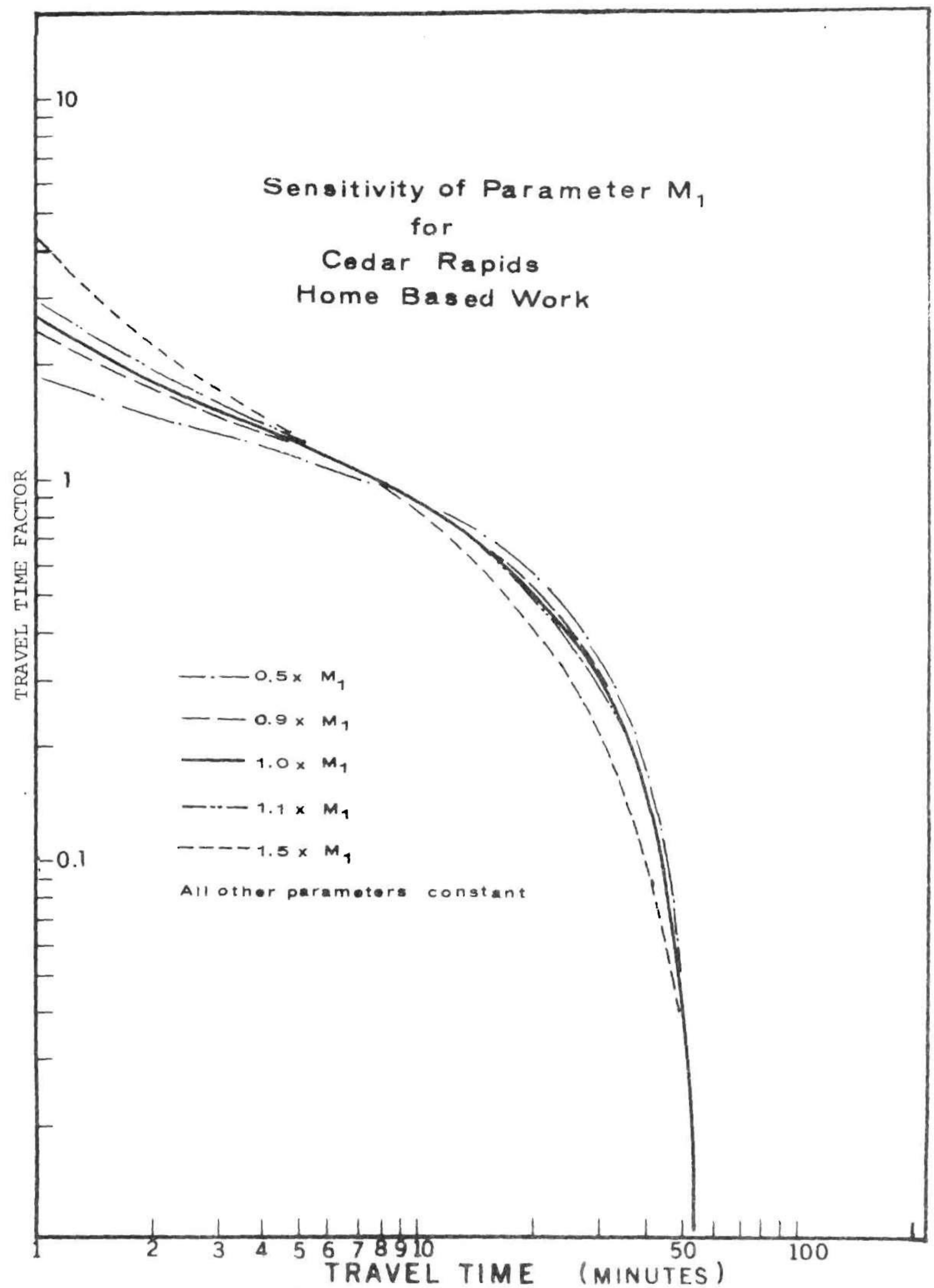


Figure 5. Sensitivity of the Pearson I parameter, m_1.

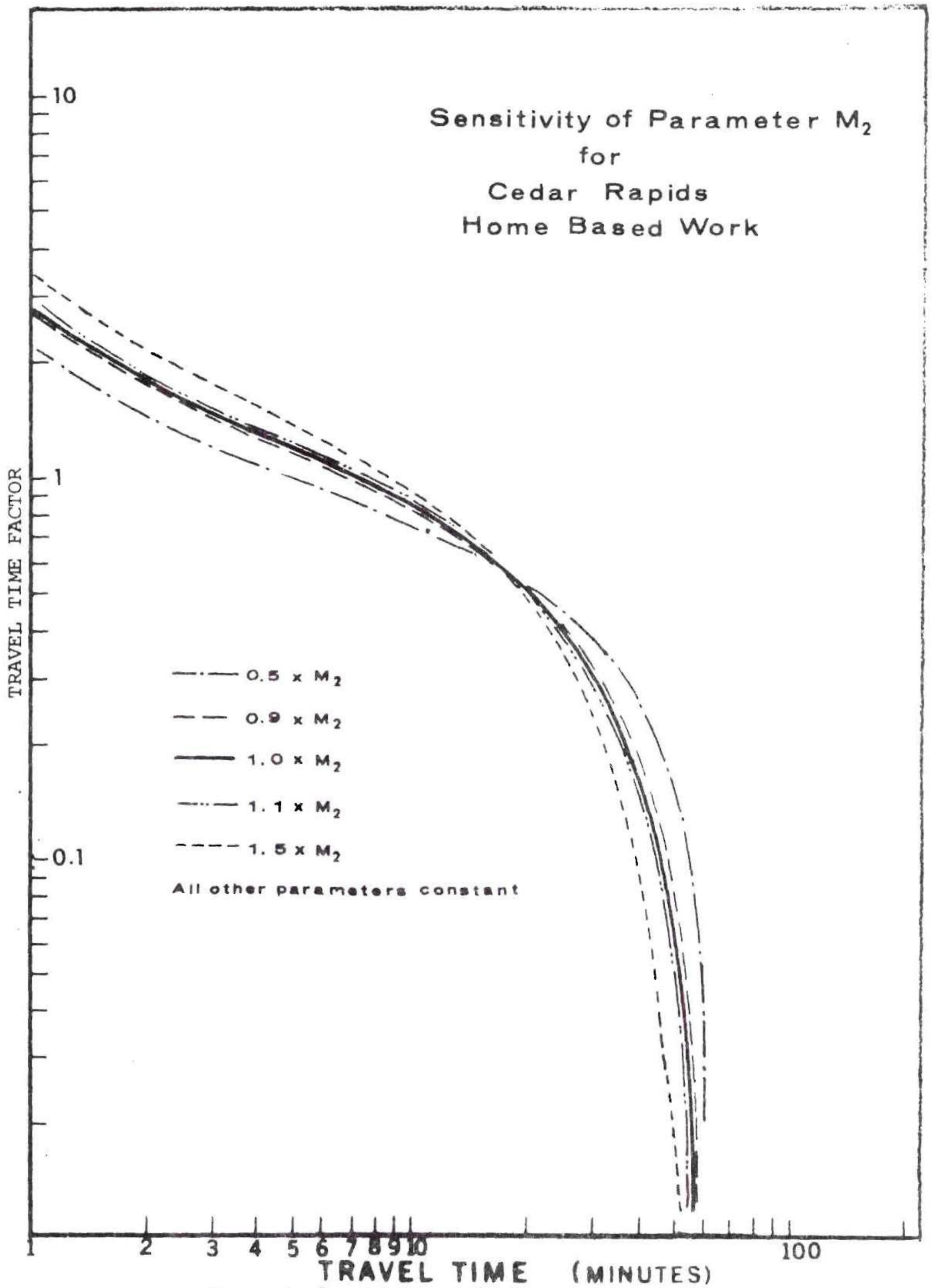


Figure 6. Sensitivity of the Pearson I parameter, m_2.

This property is obviously advantageous where there is to be any attempt to determine an estimate of a parameter from regression analysis. A highly sensitive parameter would give estimated curves highly in error, unless the standard errors of the regression equations were extremely small.

The parameter A was found to be the most sensitive of the three shape parameters, as shown by Figure 7. A 10 per cent change in value, while not radically affecting the curve shape at the low medium travel time range, did noticeably alter the curve ordinates at high travel times. The effect of a 50 per cent change in parameter value was a radical change in curve shape. The standard error of the predictive equation developed in Table 10, Appendix A, was sufficiently small that no problem of parameter sensitivity was encountered.

No sensitivity analysis was carried out for c because of the small range of values found, and the minimal effect of a change in this parameter by even the standard error of the equation. The result of change in value of c is a bodily shift of the curve along the time axis.

However, the values of all shift parameters were found to be close in absolute value. It is probable that variation in the value of c can be neglected with respect to evaluation of possible changes in the travel time factors due to changes in trip making patterns within a study area.

<u>Predictive Ability of Regression Equations</u>

Figures 8 and 9 show observed curves, and curves predicted from the regression equations.

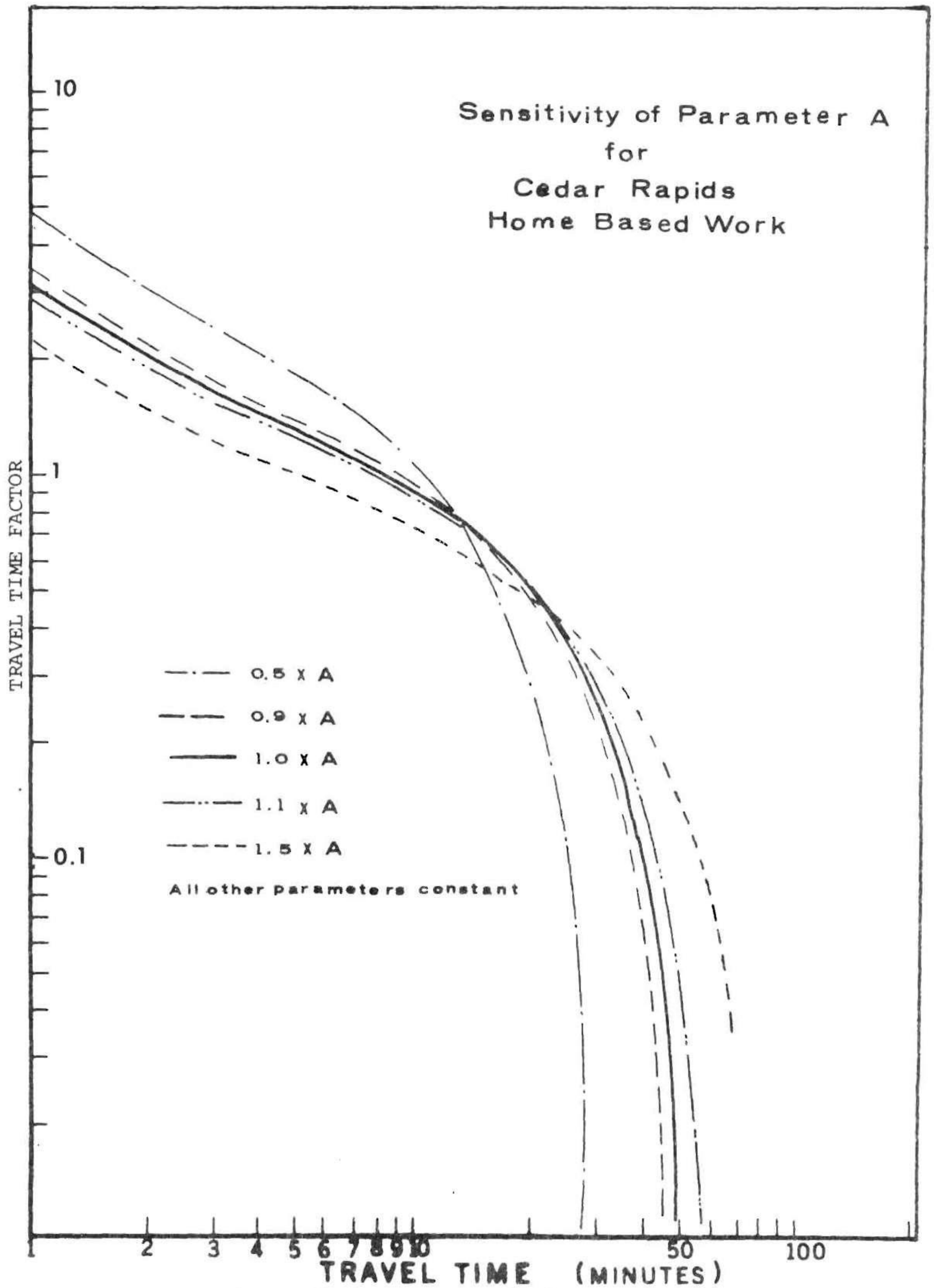


Figure 7. Sensitivity of the Pearson I parameter, A.

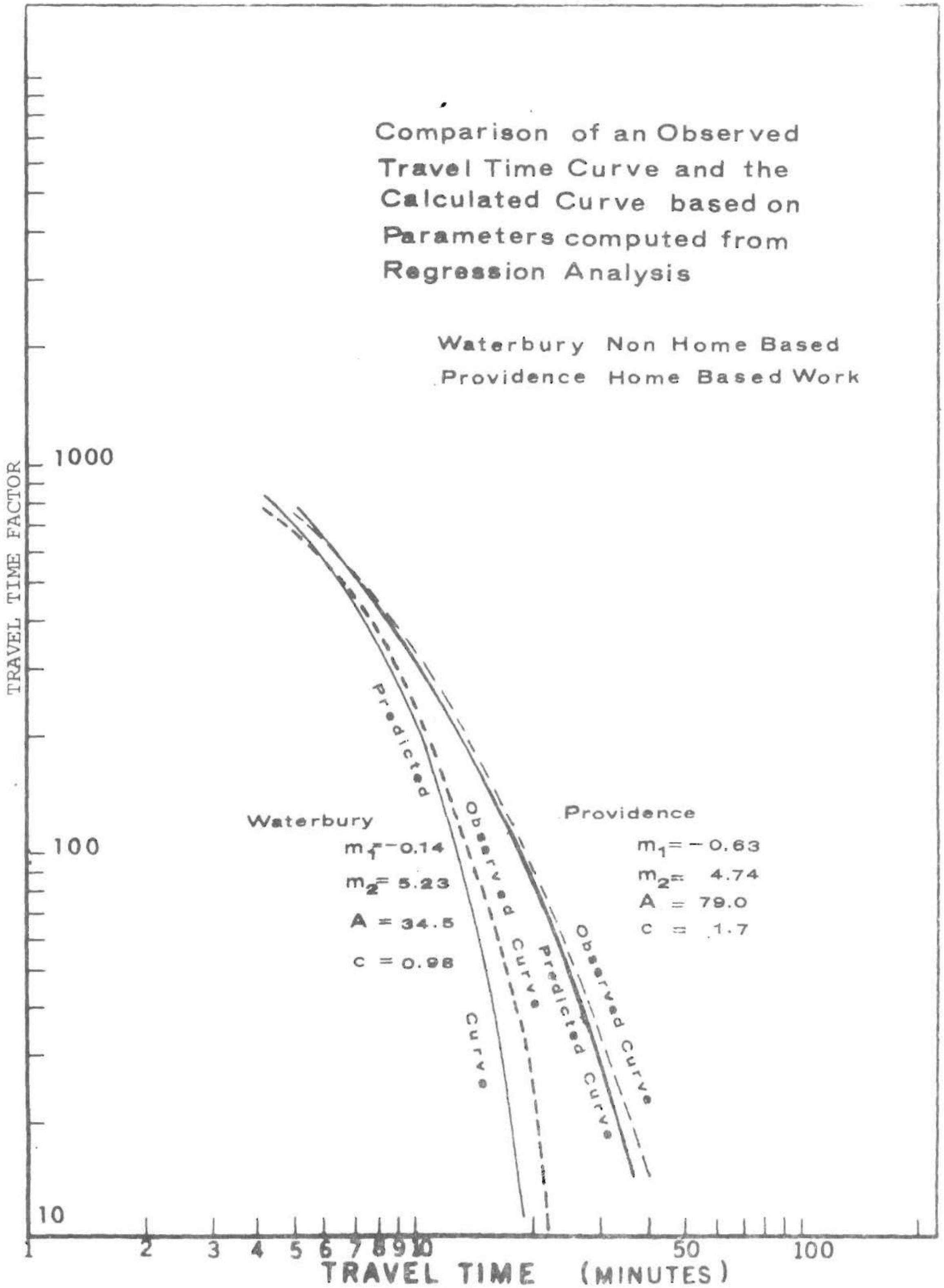


Figure 8. Predictive ability of Regression Equations for Two Pearson Type I Curves.

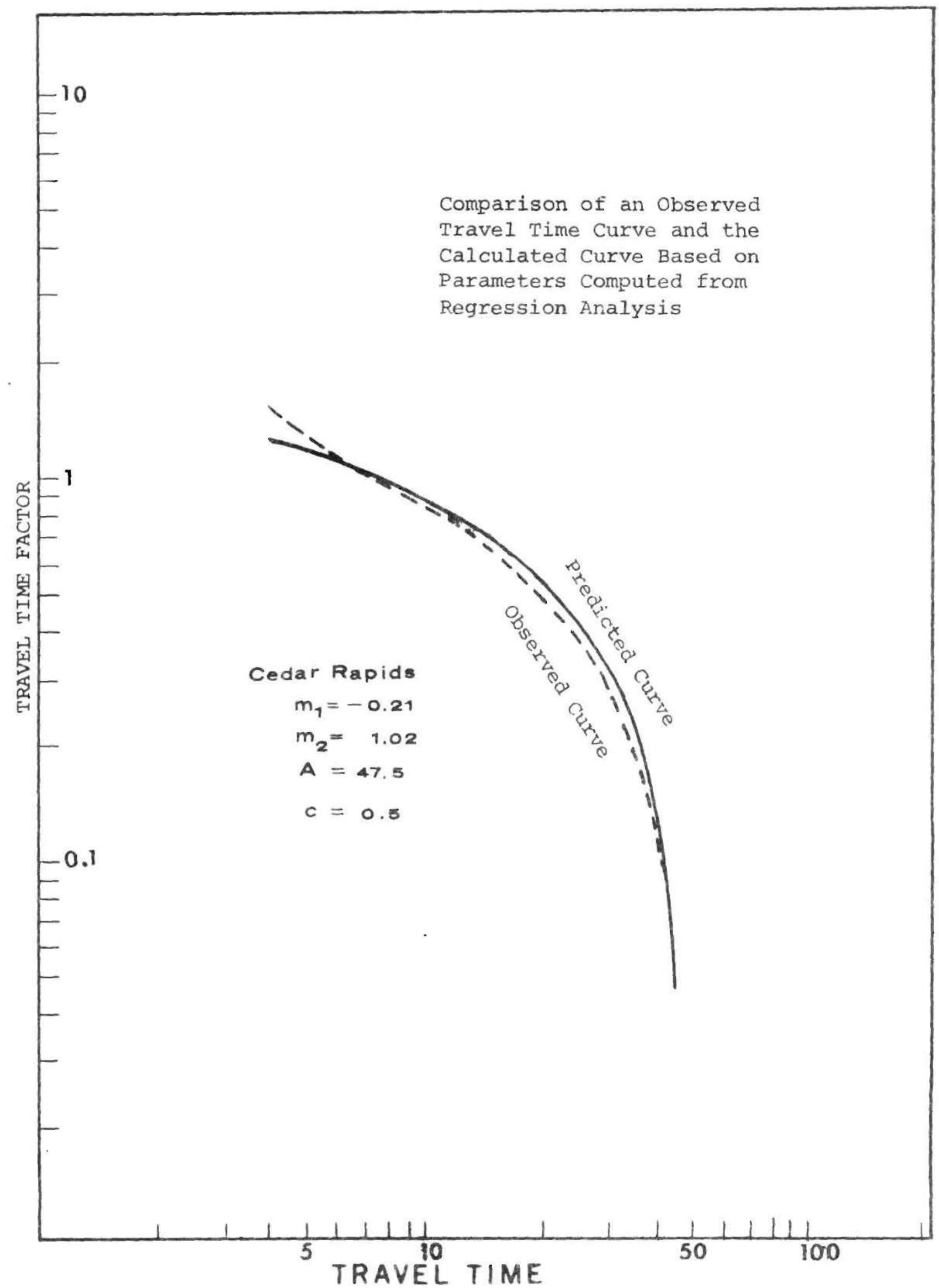


Figure 9. Predictive ability of Regression Equation for a Pearson Type I Curve.

The discovery of high sensitivity of any parameters would make the similarity in shape of observed curve and predicted curve difficult to obtain. It can be seen that the parameters predicted from city-wide variables give close estimates of true travel time curves, sufficiently close for a good first estimate of travel time curves.

Summary

A summary of the statistical findings concerning the home based work trip travel time factors would indicate:

a. Travel time factor curves can be satisfactorily modelled with Pearson Type I distribution curves.

b. The parameters of the Pearson I models are found to be statistically related to overall city-wide variables. These variables were found to be:

Number of home based work trips per thousand population.

Total number of home based work trips.

Number of trips per car.

From the statistical relationships found for home based work travel time factors, it would appear that these factors are not constant over time as is currently assumed in the calibration of the gravity models for transportation studies. Constancy over time for a particular urban area would indicate an independence of the parameters of the curve from any relationship with city variables. Any change in the trip making patterns would be assumed to have no effect on the form of the travel time curve.

A note of caution must be raised with respect to the findings of correlation of curve parameters with the city-wide variables. While the equations have been found statistically significant, the sample size involved in this study is sufficiently small that the equations involved here cannot be regarded as proven laws. These equations simply express the best relationship found between the area variables and fitted curve parameters. Further research on a larger scale would be needed to verify that these equations are not from a chance relationship. The relationships found in this work, however, should serve to raise serious questions with respect to the tenet of constancy of travel time factor over time.

Non Home Based Trips

In the case of non home based trips, homogeneous data were available for nine cities for analysis of the travel time factor curves. It was found that the most satisfactory model for the non home based trip curve was the Pearson Type I distribution, having three shape parameters and one shift parameter. The summary of results is contained in Tables 3 and 4. This curve was an accurate model over that range of travel times which included at least 90 per cent of non home based trips. A full range of travel times was therefore considered. The range of these travel times ranged from a low value of three minutes to an upper limit in the extreme case of 50 minutes. In all cases the percentage of trips falling outside the range of applicability of the model was sufficiently small that the validity of the model could not be questioned from this standpoint. As was discussed in the case of the work trip, the values of the travel time factor for very low travel times are highly question-

Table 3. Summary of Results--Non Home Based Travel Time Factors for Nine Cities

Cedar Rapids	$F(t)_1 = \frac{N_1}{38.7^{2.53}} \frac{\Gamma(3.53)}{\Gamma(0.58)\cdot\Gamma(2.95)} (t - 1.02)^{-0.42}(39.7 - t)^{1.95}$
Waterbury	$F(t)_2 = \frac{N_2}{46.7^{7.84}} \frac{\Gamma(8.84)}{\Gamma(0.82)\cdot\Gamma(8.02)} (t - 0.90)^{-0.18}(47.6 - t)^{7.02}$
Erie	$F(t)_3 = \frac{N_3}{42.6^{3.31}} \frac{\Gamma(4.31)}{\Gamma(0.42)\cdot\Gamma(3.89)} (t - 1.42)^{-0.58}(44.0 - t)^{2.89}$
Providence	$F(t)_4 = \frac{N_4}{78.3^{11.87}} \frac{\Gamma(12.87)}{\Gamma(0.39)\cdot\Gamma(12.48)} (t - 1.2)^{-0.61}(79.5 - t)^{11.48}$
Sioux Falls	$F(t)_5 = \frac{N_5}{16.6^{1.08}} \frac{\Gamma(2.08)}{\Gamma(0.46)\cdot\Gamma(1.62)} (t - 1.1)^{-0.54}(17.7 - t)^{0.62}$
Hartford	$F(t)_6 = \frac{N_6}{68.2^{8.14}} \frac{\Gamma(9.14)}{\Gamma(0.09)\cdot\Gamma(9.05)} (t - 1.07)^{-0.91}(69.3 - t)^{8.05}$
Fort Worth	$F(t)_7 = \frac{N_7}{48.0^{3.43}} \frac{\Gamma(4.43)}{\Gamma(0.32)\cdot\Gamma(4.11)} (t - 1.68)^{-0.68}(49.7 - t)^{3.11}$
Baltimore	$F(t)_8 = \frac{N_8}{109.0^{10.06}} \frac{\Gamma(11.06)}{\Gamma(0.14)\cdot\Gamma(10.92)} (t - 1.52)^{-0.86}(110.5 - t)^{9.92}$
Los Angeles	$F(t)_9 = \frac{N_9}{57.9^{1.41}} \frac{\Gamma(2.41)}{\Gamma(0.36)\cdot\Gamma(2.05)} (t - 1.3)^{-0.64}(69.2 - t)^{1.05}$

General form is Pearson I:

$$F(t)_i = \frac{N_i}{A^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1+1)\cdot\Gamma(m_2+1)} \cdot (t - c)^{m_1} \cdot (A - (t - c))^{m_2}$$

where m_1, m_2, and A are shape parameters and c is a shift parameter. N_i is a constant which modifies the magnitude of the curve ordinates, not affecting the shape or function of the travel time factors.

able due to known under-reporting of very short trips. This is especially pertinent in the case of the non home based trips. The percentage of trips falling outside the upper range of the model was also very small in all cases.

Table 4. Summary of Pearson I Shape Parameters for Non Home Based Trips

	m_1	m_2	A	c Shift Parameter
Cedar Rapids	-0.42	1.95	38.7	1.02
Waterbury	-0.18	7.02	46.7	0.90
Erie	-0.58	2.89	42.6	1.42
Providence	-0.61	11.48	78.3	1.20
Sioux Falls	-0.54	0.62	16.6	1.10
Hartford	-0.91	8.05	68.2	1.07
Fort Worth	-0.68	3.11	48.0	1.68
Baltimore	-0.86	9.92	109.0	1.52
Los Angeles	-0.64	1.05	57.9	11.3

The non home based trip is in general considerably shorter than the work trip, the range of the mean trip length ratios ranging between 60 and 70 per cent of the home based work trip. Because of the shift of the trip length distribution curve towards the shorter travel times, the fitting procedure had to be designed to obtain close fits in the region of the short travel times in excess of the minimum three minutes. This was achieved by the weighting methods described earlier under the section describing the fitting procedure.

Table 12, Appendix B, shows a comparison of the Pearson I model with the actual travel time factor. Included in this table are the indices of multiple correlation of the models to the actual curves. Also shown is the ratio of the variance of the actual travel time factors and the variance of the residuals after modelling. This ratio is expressed as an F-ratio. It is immediately apparent from a comparison of the model values and the actual values that a high degree of fit has been achieved with the use of the Pearson I curve. This is substantiated by the F-ratios which range from 1,097 to 790.9, and the indices of multiple correlation which range from 0.999 to 0.985.

Perfect fit would give values of infinity and 1.0, respectively, to the F-ratios and the indices of multiple correlation. Graphical comparison of Pearson I models and actual curves are shown in Figures 30 to 38, Appendix B.

Relationships between Curve Parameters and Area Characteristics

The second stage of statistical modelling indicated that significant relationships could be developed between the parameters of the Pearson I models and various city-wide variables. The existence of such relationships would indicate predictability, within certain expected tolerances, of change in travel time factors due to changes in the character of the city as expressed by the variables used in these statistical relationships. It was verified in all cases that the city-wide variables used in this second stage correlation were closely associated with the trip purpose in question to help remove the possibility of chance statistical significance.

A multiple regression equation was found to give the most significant model for the m_1 parameter for non home based trip models. Table 13, Appendix B, and Figure 10 indicate the form of this equation:

$$m_1 = 0.479 + 0.169 \times (\text{Total trips per car}) -$$

$$1.56 \times \frac{(\text{Non home based trips})}{\text{All trips}}$$

The multiple regression model was significant at the 2 per cent level. Partial correlation coefficients of both individual variables were 0.82, and the multiple correlation coefficient of the equation was 0.87. There was strong indication, therefore, that the value of the parameter m_1 would vary with the total trip making activity per car, and with the ratio of non home based trips to total trips in the study area.

It was observed that m_2 for non home based trip travel time factors could best be related to a multiple linear regression equation of the form shown in Table 14, Appendix B, and Figure 11:

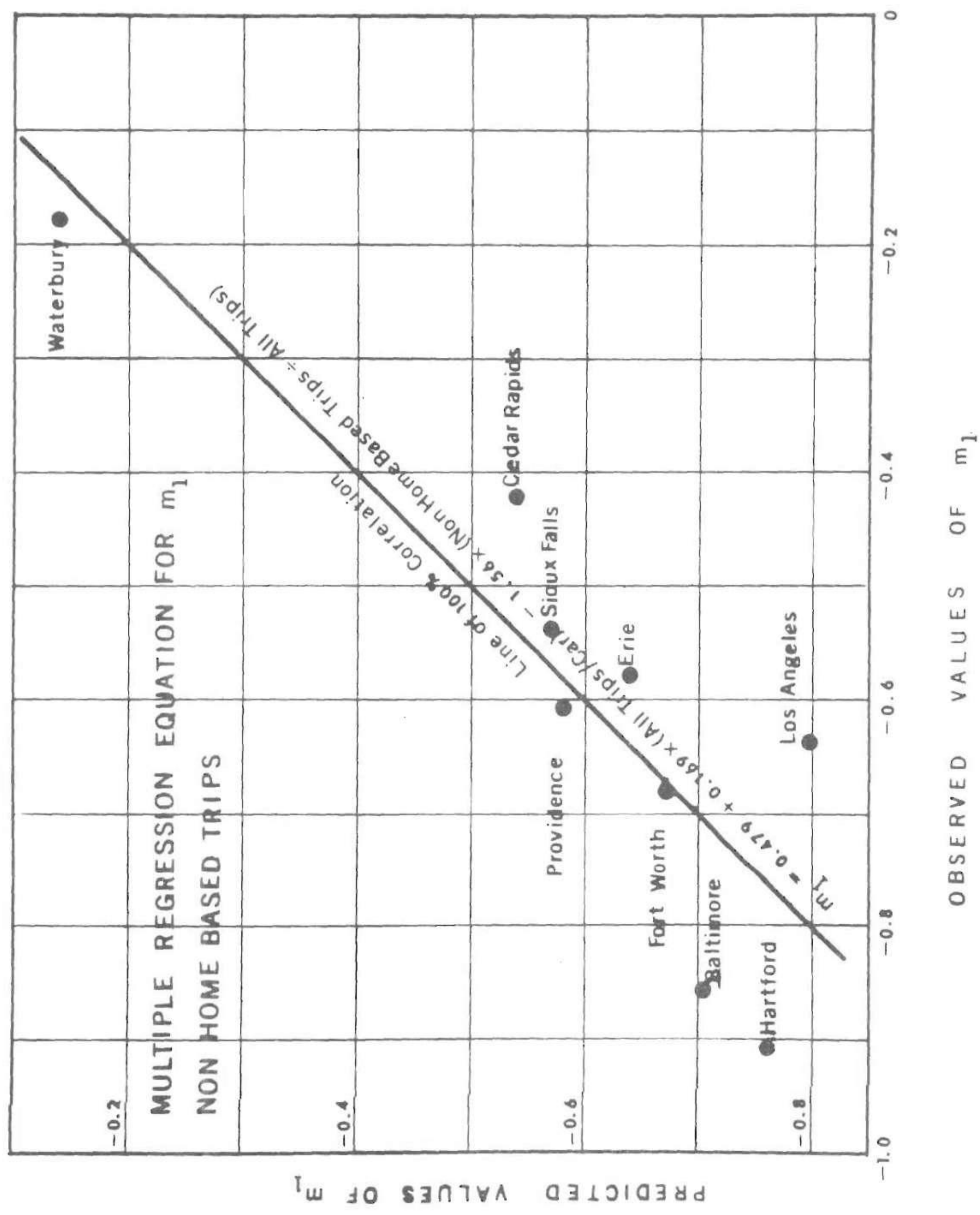


Figure 10. Regression Equation for m_1, Non Home Based Trips

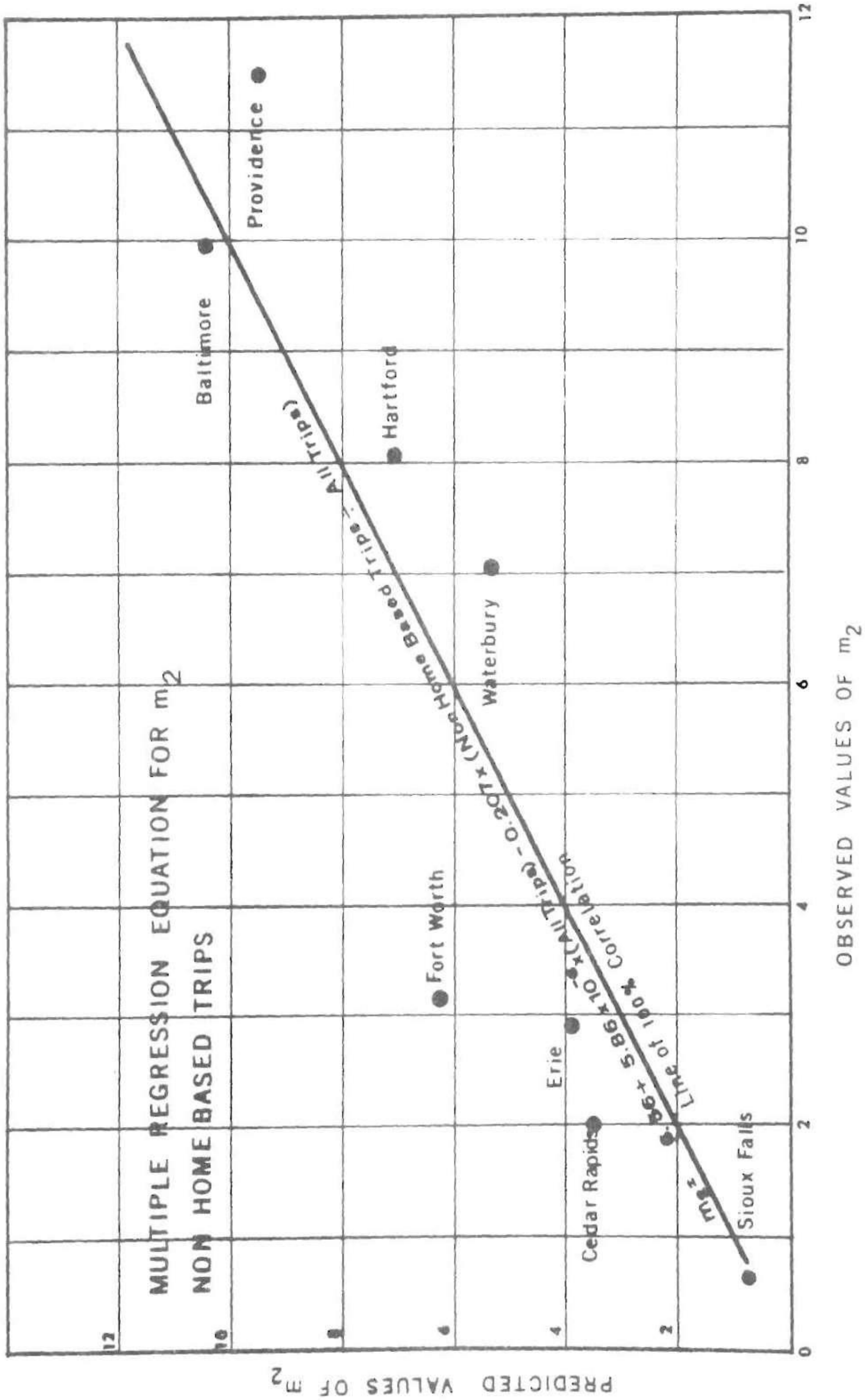


Figure 11. Regression Equation for m_2, Non Home Based Trips

$$m_2 = 6.56 + 5.86 \times 10^{-6} \times (\text{Total trips}) -$$

$$0.207 \times (\text{Non home based trips} \div \text{Total trips})$$

This equation was found to relate statistically the parameter m_2 to the total number of cars in the study area at the 2 per cent and 5 per cent significance level.

Neither m_1 or m_2 could satisfactorily be modelled with simple linear regression models by variables which had rational relationships and high correlations with these parameters.

The parameter m_2 was found to have a high relationship with the number of cars in the study area. While this is a rational variable to use to predict the parameter, the simple regression model resulting from regression analysis had a relatively high standard error. The equation, shown in Table 15, Appendix B,

$$m_2 = 2.05 + 0.0000211 \times \text{Number of Cars in the Study Area}$$

should not, however, be discounted, and might well prove to be a good model for further research.

The fitted value of m_2 for Los Angeles could not be found to correlate with any variable, nor did it follow the trends of the other values of m_2 determined for the other cities. Since the travel time curve for Los Angeles was for the major street systems network only, it was, therefore, excluded from the regression analysis. While it can also be argued that the parameters m_1 and A for Los Angeles should also be excluded from regression analysis, it was decided to include them since there appeared to be no reason on examination of Figures 10 and 12 that they could not have been reasonable estimates of the values of m_1 and A. The inclusion of these parameters in the regression analyses, while affecting the constants of the equations, made no radical change to the values. The inclusion of the m_2 value for Los Angeles obviously would have given a regression equation of questionable value, since m_2 for Los Angeles was so out of line from all other fitted values of m_2.

Table 16, Appendix B, and Figure 12 show the most satisfactory regression equation determined for the third shape parameter:

$$\ln A = 6.55 - 0.417 \ln (\text{Non Home Based Trips} \div \text{Study Area in Sq.Miles})$$

or

$$A = 699/(\text{Non Home Based Trips} \div \text{Study Area in Sq. Miles})^{0.417}$$

This equation was found to be significant at the 2 per cent level. In general, in formulating the regression equations for the parameters an attempt was made to avoid such variables as the one used here. The study area could obviously include a large amount of non-urbanized land, which would not affect the trip making activities of the area in any way. Unfortunately, insufficient information was available to enable use of trip densities with respect to urbanized land.

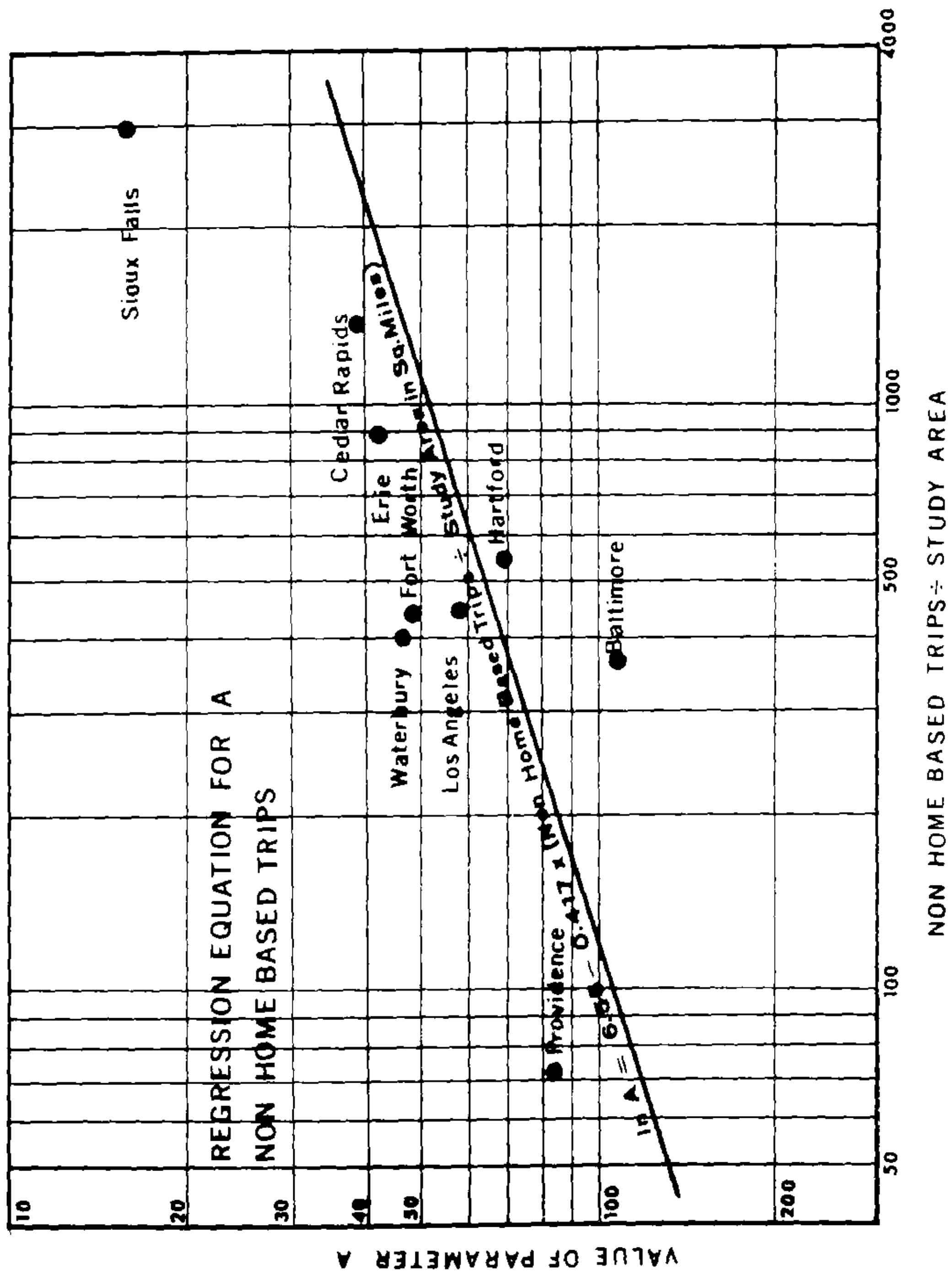


Figure 12. Regression Equation for A, Non Home Based Trips

Here and elsewhere throughout the study, knowledge of the urbanized characteristics of the area might have led to a somewhat significant breakthrough which was not possible because of sparse information of the areal characteristics.

Because no other statistically significant regression equation was available, the equation shown above should be used, but caution would be necessary where a large amount of rural land was included in a study area, causing an artificial reduction in trip making densities.

The shift parameter c for the non home based travel time curves was related in a similar way to the shape parameters to a city-wide variable. Table 13, Appendix B, and Figure 13 indicate that the shift parameter could be modelled by the following linear regression equation:

$$c \quad 1.51 - 0.17 \times (\text{Non Home Based Trips per Car})$$

This equation was significant at the 7 per cent level. This level of significance was somewhat lower than levels found elsewhere in the study. The standard error of the equation was 0.2 minutes, giving predicted values close to observed values, certainly within usable accuracy. The observed range in values of c was noted to be very small, 0.90 to 1.68. Acceptable values for estimation of c would result from the use of the mean value within this range rather than the results of the regression equation. The best equation that could be found was the equation shown in Table 17.

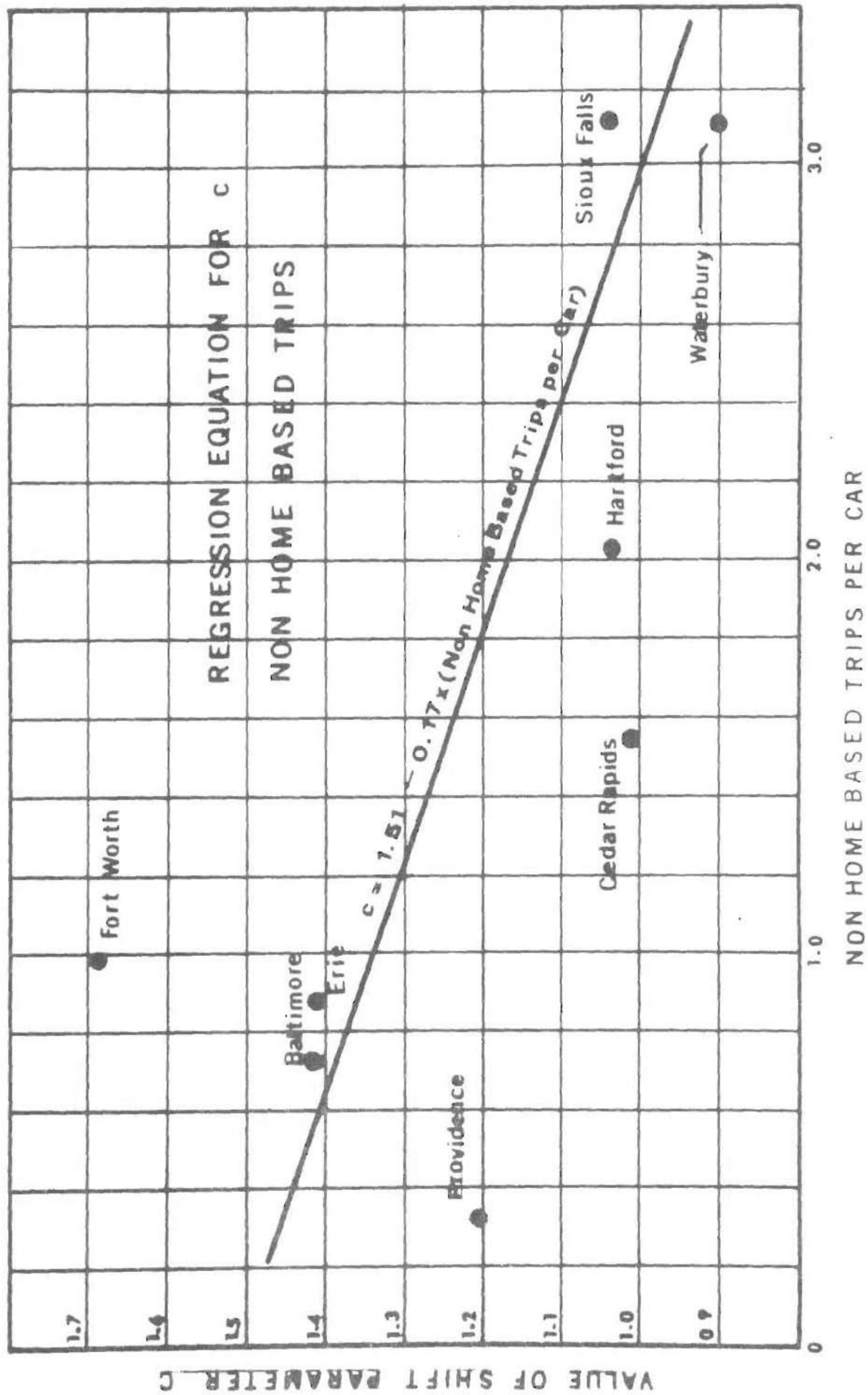


Figure 13. Regression Equation for c, Non Home Based Trips

This model was marginally significant but it is suggested that it should be used for the evaluation of c, since no better information is available, and the absolute value of the standard error of the equation was extremely small.

Sensitivity Analysis

No separate sensitivity analysis was required for the non home based models. The results of the sensitivity analysis shown in the home based work trips is applicable to Pearson I models in general. Reference is therefore made to the discussion of parameter sensitivity under the home based work trips section.

Summary

The findings on the travel time factors for the non home based trips can be summarized as follows:

a. Travel time factor curves can be satisfactorily modelled by the use of Pearson Type I distribution curves.

b. The parameters of the Pearson I models are found to be statistically related to overall city-wide variables. These variables were found to be:

All purpose trips per car

Non home based trips/all trips

Total number of cars

Non home based trips per car

Non home based trips/study area.

From the relationships determined between all curve parameters and city-wide variables, it would appear that these factors follow trends similar to those observed for work trips. The dependency of the curve parameters on independent variables would indicate that these parameters cannot be considered constant under conditions where the independent variables noted above are projected to change during the planning period. In spite of high standard errors in the equations relating the parameters to the independent variables, the statistical trends were found in general to be significant at the 2 per cent level, or better.

As was stated in relation to the work trips, caution must be exercised in the use of the regression equations developed here. The sample size of this study was relatively small, and further research would appear to be necessary to determine whether the findings can be generally applied. Further research would be greatly facilitated if information on an areal basis were more adequately collected at the time of the origin-destination study, and parametric curves used for the travel time factor.

Areal characteristics noticeably absent from some origin destination information are:

a. *Areal income:* knowledge of the distribution of income throughout the study area may well be highly significant with respect to type of travel time curve to be found in an area.

b. *Urbanized densities:* the overall density of population is a relatively meaningless figure, which can be greatly altered in a study area simply by the arbitrary inclusion of large amounts of peripheral rural areas. The population densities on urbanized land, and the distribution of these densities relative to the city center may have great impact on travel time relationships.

Shopping Trips

The final set of travel time curves analyzed were for shopping trips. Because of the various ways in which shopping trips can be classified for study purposes, this group of travel time factors presented the smallest homogeneous sample of the set. Only five travel time curves were analyzed.

At the time of analysis, an immediate difference became obvious between the shopping trip and the two other types of trip analyzed. the median shopping trip length for the cities analyzed was 7.5. This compared with 8.2 for the non home based trip and 12.2 for the work trip. Because of the short nature of the shopping trip it became essential to maximize the fit to short travel time factors. Therefore, curve fitting was heavily weighted to the short travel times.

It was found that the best fit to the shopping trip curves was obtained with the Pearson Type III curve. Figures 39 through 43, Appendix C, show graphically the fit achieved by this model. This distribution curve was found to satisfactorily model at least 90 per cent of all trips. The percentage of trips occurring above the upper limit of satisfactory fit of the model in all cases was less than 5 per cent. All models were found to fit the actual curve to times as low as two minutes. Because of the under-reporting of short trips already discussed, and the small percentage of trips for the one-minute travel time period, the models were not extended below the two-minute level.

Table 18, Appendix C, shows a comparison of the Pearson Type III model with the actual travel time factors used in the transportation studies. It will be seen that excellent fit has been obtained using this form of curve. The ratio of the variance of the unmodelled values to the residuals after regression is expressed as an F-ratio of the regression line. These F-ratios, which are an indication of fit, can be seen to be high, ranging from 760 to 2913. The index of multiple correlation, a further measure of goodness of fit can also be seen to be high, with a range from 0.990 to 0.997. Perfect fit would necessitate an infinite F-ratio, and an index of multiple correlation of 1.0. A summary of the best fit models is shown in Table 5. A summary of the shape and shift parameters is given in Table 6.

The Pearson III curve used for this type of trip, is a curve discontinuous at its lower end only. The curve is continuous to infinite travel times. The number of parameters needed to describe it is one less than for the Pearson I curve, which is discontinuous at both ends. The general form of the curve may be expressed:

$$F(t) = N \cdot \frac{p}{A} \frac{(p+1)^p}{e^{p+1}\,(p+1)} \left(1 + \frac{t-\mu}{A}\right) e^{\frac{-p}{A}(t-\mu)} ,$$

Where p and A are shape parameters.

μ is a shift parameter.

N is a constant which does not affect the shape of the curve but merely modifies the ordinate.

Table 5. Summary of Results--Shopping Trip Travel Time Factors for Five Cities

Waterbury	$F(t)_1 = N_1(0.31) \cdot \dfrac{(0.65)^{-0.35}}{e^{0.65} \cdot \Gamma(0.65)} \cdot (1 + \dfrac{t - 3.0}{-1.12}) \cdot e^{-0.31(t-3.0)}$
Erie	$F(t)_2 = N_2(0.24) \cdot \dfrac{(0.55)^{-0.45}}{e^{0.55} \cdot \Gamma(0.55)} \cdot (1 + \dfrac{t - 3.17}{-1.90}) \cdot e^{-0.24(t-3.17)}$
Providence	$F(t)_3 = N_3(0.21) \cdot \dfrac{(0.51)^{-0.49}}{e^{0.51} \cdot \Gamma(0.51)} \cdot (1 + \dfrac{t - 3.35}{-2.33}) \cdot e^{-0.21(t-3.35)}$
Fort Worth	$F(t)_4 = N_4(0.40) \cdot \dfrac{(0.61)^{-0.39}}{e^{0.61} \cdot \Gamma(0.61)} \cdot (1 + \dfrac{t - 2.30}{-0.97}) \cdot e^{-0.40(t-2.30)}$
Hartford	$F(t)_5 = N_5(0.19) \cdot \dfrac{(0.21)^{-0.79}}{e^{0.21} \cdot \Gamma(0.21)} \cdot (1 + \dfrac{t - 1.99}{-4.27}) \cdot e^{-0.19(t-1.99)}$

General form of the Pearson Curve III:

$$F(t)_i = N_i \cdot \frac{p}{A} \quad \frac{(p + 1)^p}{e^{p+1} \quad (p + 1)} \cdot (1 + \frac{t - \mu}{A})^p \cdot e^{-p/A \cdot (t-\mu)}$$

Where N_i is a constant which merely modifies the magnitude of the curve ordinates but does not affect the shape or function of the travel time factors. p and A are the shape parameters, and μ the shift parameter of the Pearson III distribution.

Table 6. Summary of Pearson III Shape Parameters for Home Based Shopping Trips

	p	A	μ Shift Parameter
Waterbury	-0.35	-1.12	2.85
Erie	-0.45	-1.90	3.16
Providence	-0.49	-2.33	3.32
Hartford	-0.79	-4.27	1.91
Fort Worth	-0.39	-0.97	2.54

The second stage of correlation for the shopping trip curves therefore amounted to the relation of the two shape parameters p and A, and the shift parameter μ to the city-wide variables. It was found that a close statistical relationship could be established between city-wide variables and the curve parameters. In most cases a relation was established at the 1 per cent level, with no relation being significant at a level greater than 2 per cent.

Relationship Between Curve Parameters and Area Characteristics

Table 24, Appendix D, indicates the correlation coefficients between the curve parameters (p,A and μ) and various city-wide variables. Based on these correlation coefficients, a regression analysis was made on those variables which had the highest correlation coefficients, and also seemed most useful as predictive variables. Both shape parameters, p and A, were related strongly to the ratio of home based other than work trip trips to total trips. The shift parameter u was strongly related to car ownership.

It was found that the strong relationship between the shape parameter A and the ratio of home based other than work trips to total trips could be expressed in the form shown in Table 19, Appendix C:

$$A = 12.74 - 22.7 \times (\text{Home Based Other Than Work Trips} \div \text{Total Trips})$$

This regression equation was significant at the 2 per cent level, indicating a statistical probability that as the ratio of home-based other than work trips increases, the value of the shape parameter A can be expected to decrease.

An equally significant relationship was determined between this ratio of home based other than work trips to total trips and the second shape parameter p. This relationship is shown by the correlation coefficient of 0.97 between the two values for each city, as shown in Table 24, Appendix D. Table 20, Appendix D, displays the results of the regression analysis on p:

$$p \quad -1.92 + 0.0305 \times (\text{Home Based Other Than Work Trips} \div \text{Total Trips})$$

This equation was found to be significant at the 1 per cent level. Statistically, therefore, there was found to be a strong indication that the absolute value of p could be expected to decrease as the ratio of home based other than work trips to total trips increased. It should be noted that p will always be negative for this shape of curve.

While the variable chosen as a predictive variable for both A and p in Table 19 and Table 20, is a reasonable choice, the correlation coefficient and the linear regression models were highly dependent on the Hartford parameters since these differed a great deal from the other four. The equations were therefore regarded as suspect, requiring an expansion of sample size before acceptance. Alternative equations based on other variables with a better distribution of points along the graph are shown in Figures 14 and 15. These regression equations were significant only at the 10 per cent level. However, in both cases, the standard error of these equations was lower than with the variables used in Tables 19 and 20. While the equations:

$$A = 12.74 - 22.7 \times (\text{Home Based Other Than Work Trips} \div \text{Total Trips})$$

$$p = -1.92 - 0.0305 \times (\text{Home Based Other Than Work Trips} \div \text{Total Trips})$$

cannot be disregarded, it is felt that with the small sample size used in determining these equations, it is better to use:

$$\ln. (A) = 1.37 + 1.28 \times 10^{-3} \times \text{Total Trips per 1000 Population}$$

$$\ln. (p) = 1.11 + 5.10 \times 10^{-7} \times \text{Total Trips}$$

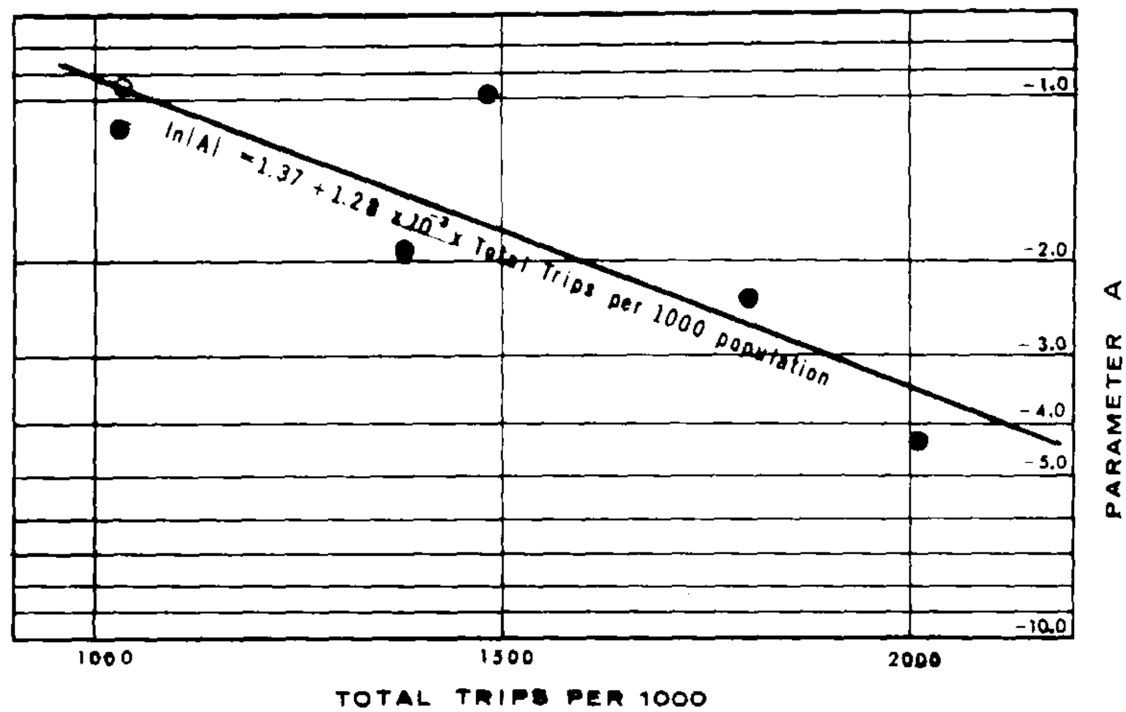


Figure 14. Regression Equation for A, Shopping Trips

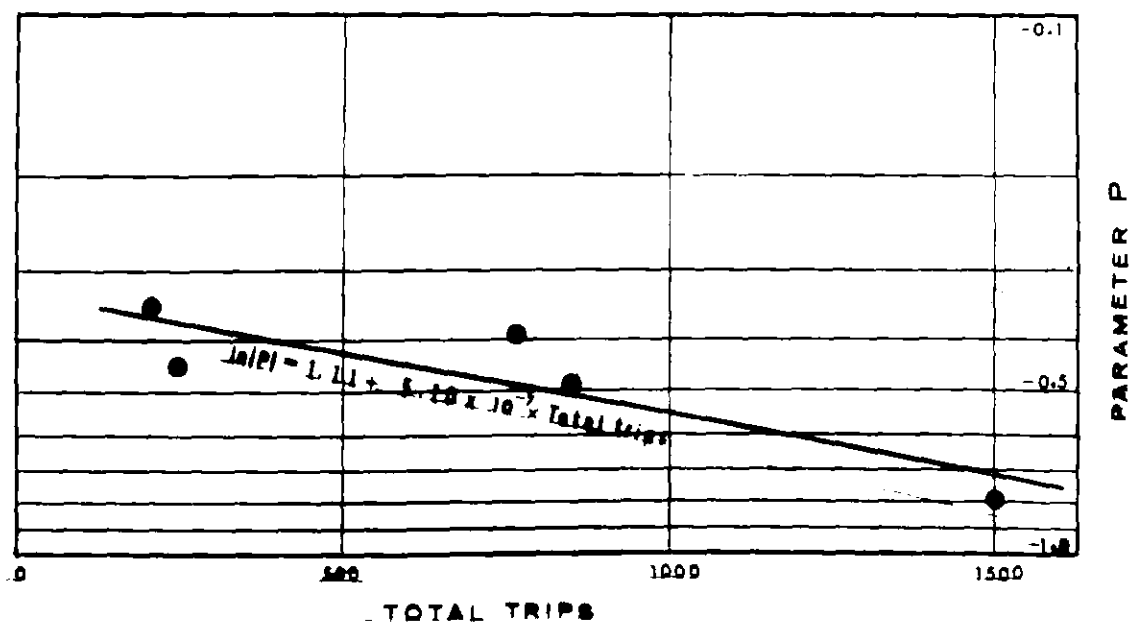


Figure 15. Regression Equation for p, Shopping Trips

It should also be noted that with latter two equations, the values of p and A cannot assume impossible values, which is possible with the equations in Tables 19 and 20. For the shapes of curve encountered with the travel time factor, p and A will always be negative.

The final regression analysis carried out was on the calculated value of μ, which besides being a shift parameter is in fact the mean of the distribution. This mean value was found to be closely related to the variable which best described the intensity of car ownership in the area, i.e. cars per person. The regression equation which best described this relationship was the form shown in Table 21, Appendix C, and Figure 16:

$$\mu = 3.06 - 15.79 \times (\text{Cars per Person})$$

This equation was found to be significant at the 1 per cent level, indicating high statistical probability that as the intensity of car ownership increases the mean value of the travel time curve distribution decreases.

<u>Sensitivity Analysis</u>

The sensitivity of p is shown in Figure 17. Graphs showing the effect of a reduction and an increase of p by 10 and 50 per cent is shown. A reduction in p results in an effective reduction in the overall slope of the curve. For small changes in p, a small change is noted at both the upper and lower ranges of travel time. The parameter is not highly sensitive to small changes, but large changes in value give radically changed forms of curve.

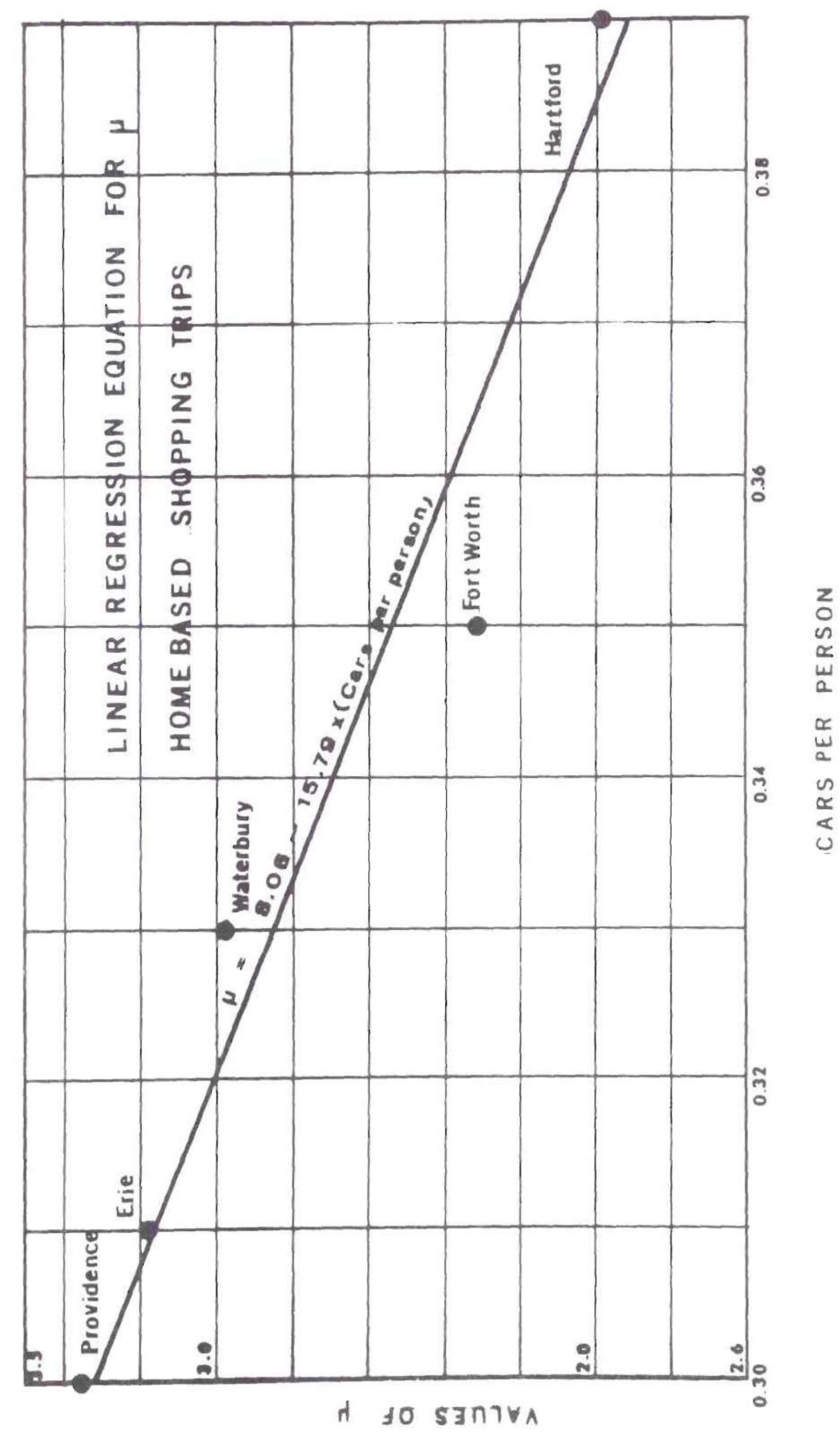


Figure 16. Regression Equation for μ, Shopping Trips

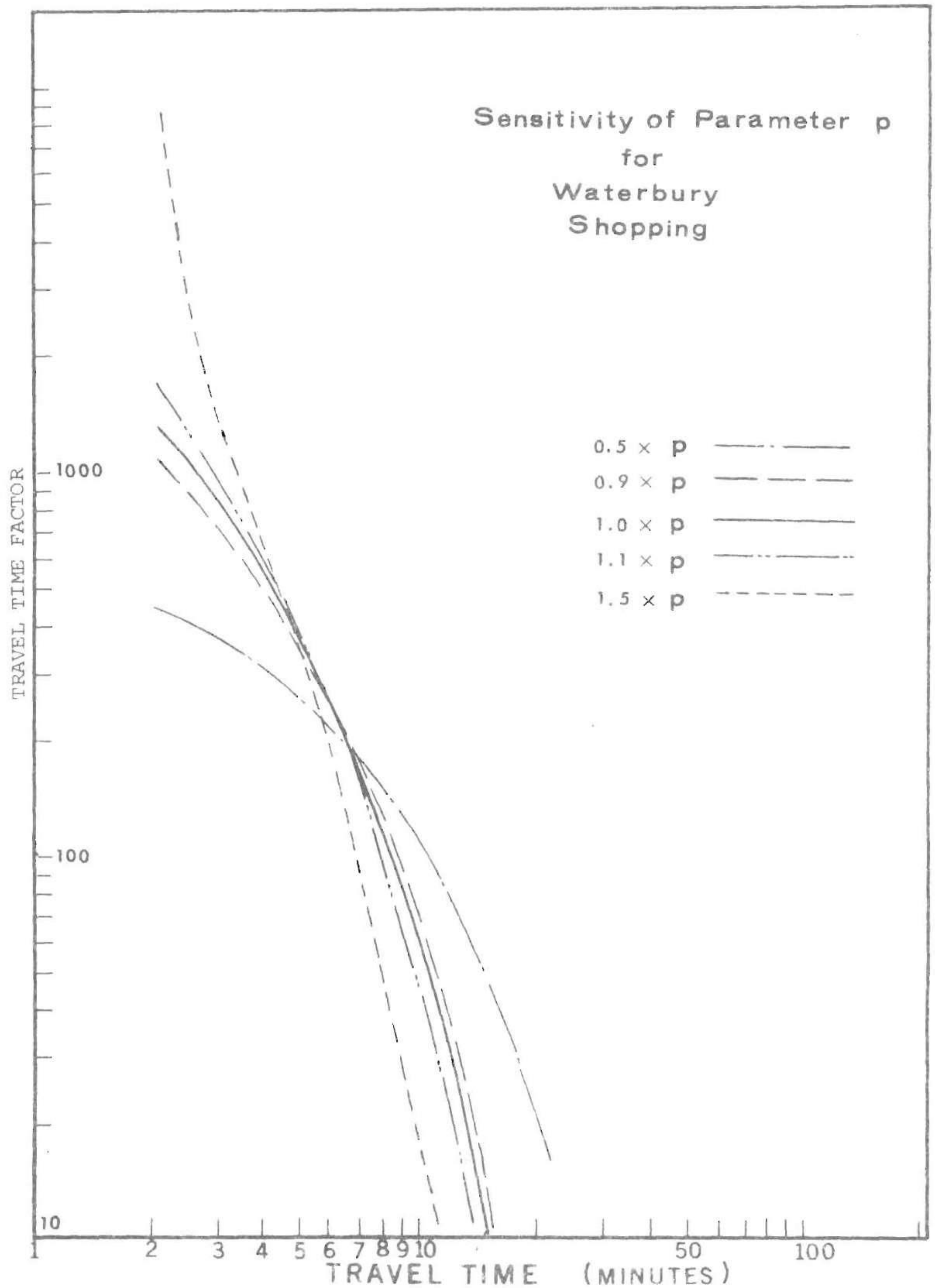


Figure 17. Sensitivity of Pearson III Parameter p

The sensitivity of A is depicted in Figure 18. It would appear that this parameter is more sensitive to change at the lower range of travel time than p, but is still not too sensitive for use in modelling.

The ability of the regression analyses to predict a travel time curve is shown in Figure 19. The agreement between the curve determined from the regression parameters and the actual travel time curve is close. However, it must be noted that the sample size involved in working with the shopping trips was so small that the great differences are unlikely.

Summary

A summary of the statistical findings concerning shopping trip travel time factor curves would indicate that:

a. Travel time factor curves can be satisfactorily modelled using Pearson Type III distribution curves.

b. The parameters of the Pearson Type III curves which best fit the actual curves, were found to be statistically related to city-wide variables. These variables were found to be:

Ratio of home based other than work trips to total trips.

Car ownership per person.

From the statistical relationship determined for the parameters of the shopping trip models, it would appear that the current assumption of constancy of travel time factors is incorrect. Constancy of the travel time factor would necessitate independence of the parameters of a suitable model from any of the factors which make up the transportation background of the city. Under this assumption any change in the ratio of home-based other than work to total trips would have no effect on

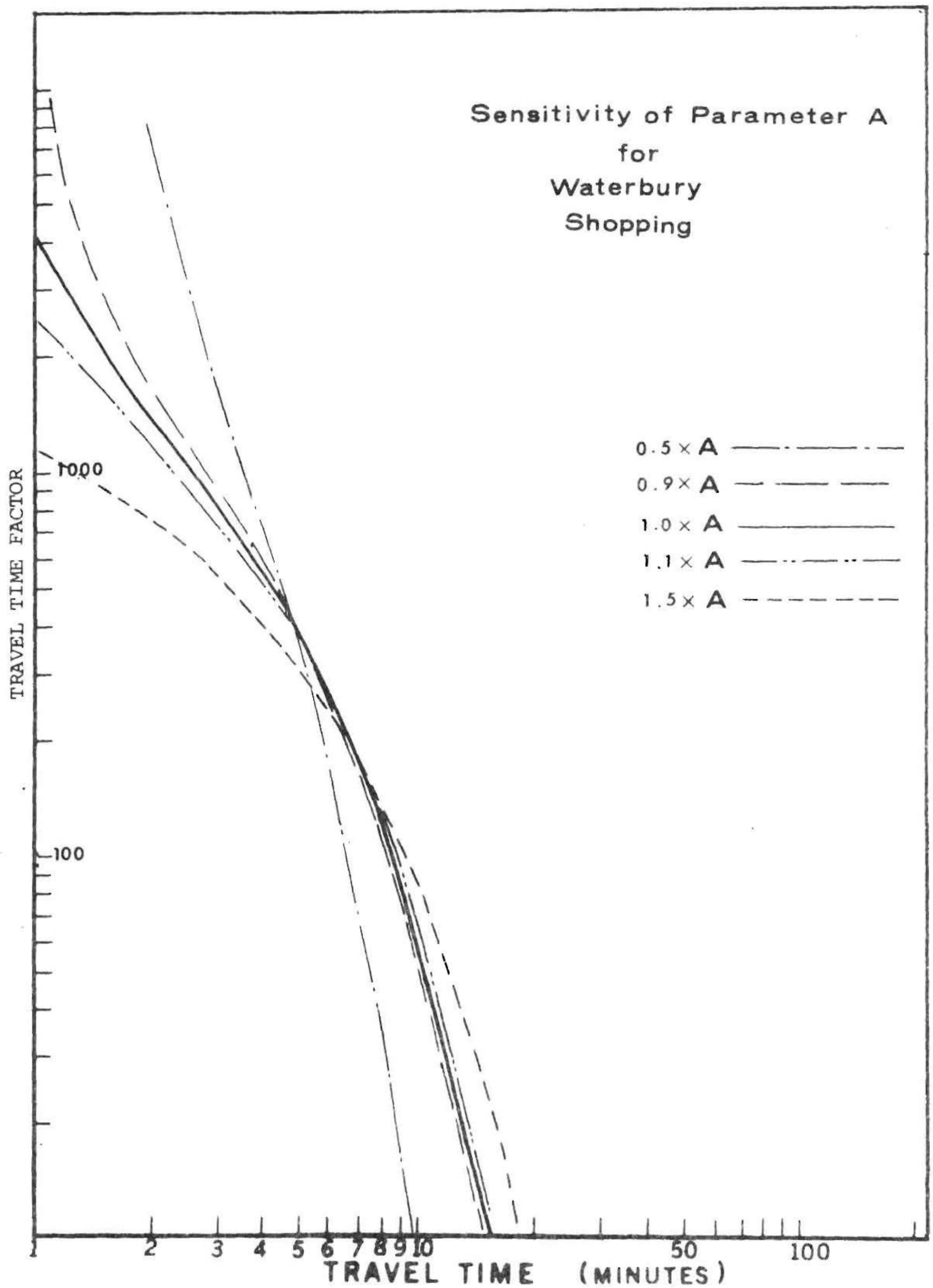


Figure 18. Sensitivity of Pearson III Parameter A

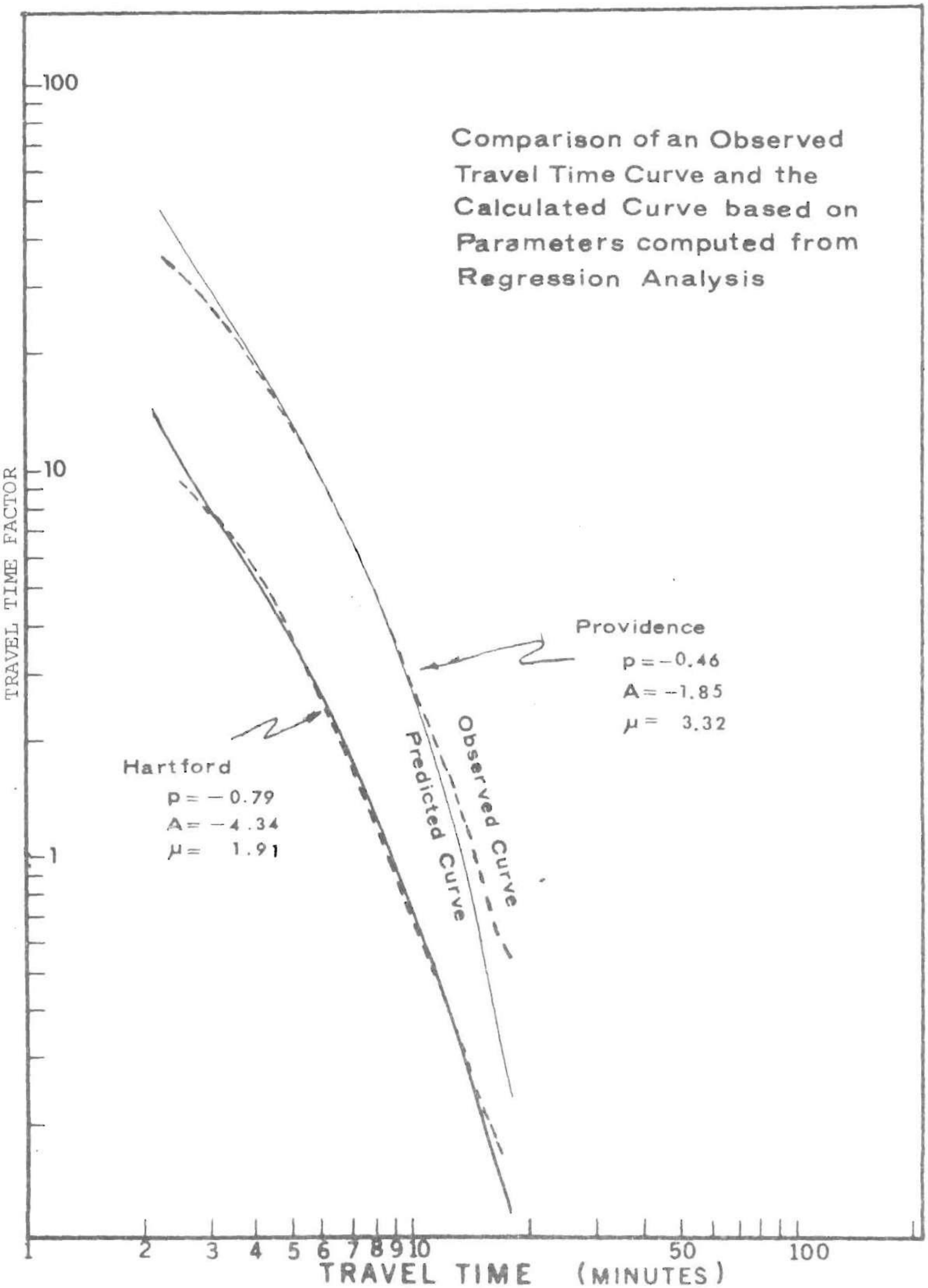


Figure 19. Predictive Ability of Regression Equations for Pearson III Curves

travel time factors. The finding here would indicate that this hypothesis, which is not appealing to rational consideration, is also not correct. Any change in this ratio would indicate that basic differences in travel characteristics have taken place. The regression equations developed indicate a method whereby the change in travel time curve parameter for a given change in travel behavior can be predicted.

CHAPTER V

CONCLUSIONS

1. Empirical travel time factor curves for transportation studies can be closely approximated by parametric curves of the Pearson system. Parametric curves of similar general properties as the travel time factor can be computed. Close fit can be obtained by equating the moments of empirical and theoretical curves. The Pearson system of curves satisfy the theoretical requirements of the travel time factor with respect to decay with time better than other suggested distributions.

2. Home based work and non home based travel time curves are best modelled by Pearson Type I curves. Pearson Type III curves were found to provide a better model for the shopping trip which has a lower mean trip time than home based work or non home based trips. It would appear that the Pearson I distribution provides a better fit to those travel time curves where the change of time exponent with time is most apparent.

3. Statistical relationships were found between the parameters of the Pearson models and pertinent city-wide variables. Among those variables which were related to model parameters were:

Home based work trips per thousand population

Total trips per car

Total home based trips

Ratio of non home based trips to all trips

Total trips in the study area

Non home based trips/study area

None home based trips/car

Cars per person

Ratio of home based other than work trips to total trips.

4. Significant regression equations between study area variables and model parameters can be calculated, indicating that there is statistical probability that these parameters are not constant, but are likely to change as the character of the area itself is modified. Such modification would be reflected in change in the area-wide variables.

5. While the relationships between area variables and curve parameters can be expressed by regression equations, the travel time curve for any particular city is not likely to be predictable with satisfactory accuracy, since the standard error of all regression equations was high compared to the sensitivity of the parameters.

6. The regression equations can be used without serious error for a first approximation of the travel time factors Modification of the initial estimate of travel time curves can be effected in a manner similar to the method suggested in the Bureau of Public Roads Manuals (4,7), except that the new curves should be computed by moments, rather than the hand-fitting method currently recommended. This would enable the formation of a data bank of mathematical expressions for travel time curves and study area characteristics so that "meaningful comparisons can be made between these expressions for different urban areas with various population and density characteristics" (4). This curve fitting

technique is easily programmed for high-speed computers, and will speed the present gravity model iterative fitting techniques.

7. The Pearson I and III shape parameters were not found highly sensitive. Small errors or small changes in parameters did not give radically different curves. Highly sensitive parameters, indicating a relatively unstable model, are undesirable.

CHAPTER VI

RECOMMENDATIONS

1. Hand drawn travel time curves should be abandoned where parametric curves can be found to give an adequate fit for the Gravity Model Trip distribution procedure. The present empirical approach to the travel time factor inhibits correlation of the effects of city characteristics on travel time from city to city. Fits of model derived trip distribution curves to origin destination curves could be obtained by the use of parametric curves, with the accuracy limits currently achieved by empirical methods. It is suggested that the Pearson system of curves may well serve as an adequate parametric system.

2. Many important study area variables which are currently neglected on a wide scale in transportation studies, should be collected. Income distribution, population density distribution, and expressway and freeway mileage and distribution for example could be fairly easily assembled at the time of the origin destination study. These basic measures of level of consumer affluence, consumer demand and facility service levels which were unavailable in this research, may well be better measures of the characteristics of a city's transportation network than those used in this study. A great deal more attention should be focused on the areal characteristics rather than simply an assembly of zonal characteristics. This latter approach neglects the effect of

arrangement of zonal land use. The spatial arrangement of zones greatly affects the behavior of the travel time factor, which is areal not zonal in its formulation.

3. The sample size used in this research was regrettably small. The work should be expanded to a larger scale project, using in lieu of driving time models, total travel time which has been suggested is a better measure of spatial separation.

4. It was beyond the scope of this research to actually compute a gravity model distribution of interzonal trips using travel time factors derived from a parametric fit. The effect of the use of parametric curves should be investigated to determine whether any serious problems occur at the very long travel times, i.e. the longest 5 per cent of trips. All parametric models underestimated the travel time curves at these travel times. While the percentage error was high, the actual effect on zonal distributions could only be determined by a comparison of parametric distribution and origin destination data. This research would indicate the validity of the Pearson models at long travel times.

APPENDIX A

HOME BASED WORK MODELS

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work

CEDAR RAPIDS--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model	Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	1.50	1.37	25	0.39	0.37
5	1.25	1.24	26	0.37	0.35
6	1.10	1.14	27	0.35	0.33
7	1.00	1.06	28	0.33	0.31
8	0.93	0.98	29	0.31	0.29
9	0.87	0.92	30	0.29	0.28
10	0.84	0.86	31	0.27	0.26
11	0.80	0.81	32	0.25	0.24
12	0.76	0.76	33	0.23	0.22
13	0.72	0.73	34	0.21	0.21
14	0.68	0.69	35	0.19	0.19
15	0.64	0.65	36	0.17	0.18
16	0.61	0.61	37	0.15	0.16
17	0.58	0.58	38	0.14	0.15
18	0.55	0.55	39	0.13	0.14
19	0.52	0.52	40	0.12	0.12
20	0.49	0.49	41	0.11	0.11
21	0.47	0.47	42	0.10	0.10
22	0.45	0.44	43	0.09	0.09
23	0.43	0.42	44	0.08	0.08
24	0.41	0.40	45	0.06	0.07

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 13,861.0

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)

WATERBURY--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	700	705
5	540	511
6	425	397
7	340	321
8	282	266
9	240	225
10	204	193
11	172	166
12	146	145
13	123	127
14	106	111
15	93	98
16	83	87
17	75	77
18	68	69
19	61	61
20	55	55
21	49	49
22	43	44
23	38	39
24	34	35
25	30	31
26	27	28
27	25	25
28	23	22
29	21	20
30	19	18
31	18	16
32	17	14
33	16	12
34	15	11
35	14	10
36	13	9

Index of Multiple Correlation = 0.995
F-Ratio of Regression = 2593.1

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
ERIE--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	5900	6174
4	5000	4971
5	4200	4211
6	3700	3600
7	3300	3230
8	2900	2880
9	2650	2585
10	2350	2331
11	2100	2108
12	1860	1910
13	1700	1733
14	1540	1573
15	1400	1428
16	1260	1295
17	1160	1174
18	1130	1062
19	920	960
20	850	865
21	760	777
22	680	696
23	610	621
24	540	551
25	490	488
26	430	429
27	390	374
28	340	324
29	300	279
30	270	237

Index of Multiple Correlation = .999
F-Ratio of Regression = 26,732.9

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
NEW ORLEANS--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model	Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	540	573.2	26	29	31.4
5	425	411.9	27	27	28.6
6	335	321.1	28	25	26.1
7	278	261.9	29	23	23.8
8	230	219.7	30	21	21.6
9	195	187.9	31	19	19.7
10	166	163.0	32	18	17.9
11	142	142.9	33	17	16.3
12	123	126.3	34	16	14.8
13	109	112.3	35	14	13.4
14	98	100.4	36	13	12.1
15	86	90.2	37	12	10.9
16	77	81.3	38	11	9.9
17	69	73.4	39	11	8.9
18	62	66.5	40	10	8.0
19	56	60.3	41	9	7.2
20	50	54.8	42	9	6.4
21	45	49.9	43	8	5.7
22	42	45.4	44	7	5.1
23	38	41.4	45	7	4.5
24	35	37.7	46	6	4.0
25	32	34.4			

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 6288.5

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
PROVIDENCE--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model	Travel Time (Minutes)	Travel Time Factor	Pearson I Model
5	720.0	766.7	28	42.0	43.9
6	600.0	592.4	29	38.0	40.0
7	510.0	480.2	30	35.0	36.4
8	450.0	400.9	31	32.2	33.1
9	390.0	341.3	32	29.5	30.1
10	320.0	294.7	33	27.3	27.4
11	280.0	257.2	34	25.0	24.9
12	245.0	266.3	35	23.0	22.7
13	215.0	200.3	36	21.0	20.6
14	185.0	178.3	37	19.5	18.7
15	160.0	159.3	38	18.0	17.0
16	145.0	142.9	39	16.5	15.4
17	125.0	128.5	40	15.0	14.0
18	112.0	115.8	41	14.0	12.7
19	100.0	104.6	42	13.0	11.5
20	90.0	94.6	43	12.0	10.4
21	82.5	85.7	44	11.0	9.4
22	72.5	77.8	45	10.1	8.5
23	67.0	70.6	46	9.2	7.6
24	60.0	64.1	47	8.4	6.9
25	55.0	58.3	48	7.6	6.2
26	51.0	53.0	49	7.1	5.6
27	46.0	48.3	50	6.6	5.0

Index of Multiple Correlation = 0.992
F-Ratio of Regression = 2434.3

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
SIOUX FALLS--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
2	210	225.3
3	185	172.0
4	150	144.4
5	125	125.7
6	110	111.5
7	100	99.9
8	85	90.0
9	79	81.1
10	67	72.8
11	61	65.0
12	57	57.4
13	50	49.7
14	48	41.7
15	45	32.7
16	10	21.3

Index of Multiple Correlation = 0.979
F-Ratio of Regression = 831.9

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
HARTFORD--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	6.58	6.60
4	5.02	4.79
5	3.99	3.77
6	3.20	3.09
7	2.75	2.60
8	2.18	2.23
9	1.89	1.93
10	1.55	1.69
11	1.40	1.49
12	1.20	1.32
13	1.04	1.17
14	0.98	1.05
15	0.84	0.94
16	0.78	0.84
17	0.72	0.75
18	0.67	0.68
19	0.62	0.61
20	0.57	0.55
21	0.53	0.49
22	0.49	0.44
23	0.45	0.40
24	0.41	0.36
25	0.37	0.32
26	0.33	0.29
27	0.29	0.26
28	0.25	0.23
29	0.21	0.21
30	0.17	0.18
31	0.14	0.16
32	0.12	0.14
33	0.10	0.12
34	0.09	0.11
35	0.08	0.10
36	0.08	0.09
37	0.07	0.08
38	0.07	0.07
39	0.06	0.06
40	0.06	0.05

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 5775.6

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)
FORT WORTH--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	288.0	274.9
5	202.0	183.1
6	138.0	134.0
7	101.0	103.2
8	77.0	82.0
9	62.0	66.6
10	50.0	54.9
11	41.0	45.8
12	35.0	38.5
13	30.0	32.6
14	26.0	27.7
15	23.0	23.7
16	21.0	20.3
17	18.0	17.4
18	17.0	14.9
19	15.0	12.8
20	14.0	11.0
21	13.0	9.5
22	12.0	8.1
23	11.0	7.0
24	10.0	6.0
25	9.0	5.1
26	9.0	4.3
27	9.0	3.7

Index of Multiple Correlation = 0.993
F-Ratio of Regression = 1378.6

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)

BALTIMORE--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model	Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	655	757.7	27	44	46.6
5	655	566.9	28	40	42.4
6	442	454.4	29	34	38.5
7	373	378.6	30	30	35.0
8	325	323.2	31	27	31.7
9	268	280.5	32	27	28.6
10	235	246.4	33	24	25.8
11	201	218.4	34	24	23.2
12	192	195.0	35	23	20.8
13	186	175.0	36	21	18.6
14	151	157.7	37	20	16.6
15	142	142.6	38	18	14.7
16	128	129.3	39	18	13.0
17	110	117.5	40	13	11.4
18	107	106.9	41	12	10.0
19	99	97.4	42	7	8.7
20	96	88.8	43	7	7.5
21	93	81.0	44	7	6.4
22	77	74.0	45	6	5.4
23	65	67.5	46	3	4.6
24	63	61.6	47	3	4.6
25	64	56.2	48	2	3.1
26	53	51.2	49	4	2.5

Index of Multiple Correlation = 0.988
F-Ratio of Regression = 1701.9

Table 7. Comparison of Travel Time Factors with Pearson I Models--Home Based Work (Continued)

LOS ANGELES--Home Based Work

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
13	45.0	46.9
14	34.0	31.1
15	27.0	23.0
16	20.0	18.0
17	16.0	14.6
18	13.2	12.1
19	11.0	10.2
20	9.0	8.7
21	8.0	7.5
22	6.5	6.5
23	5.4	5.7
24	4.8	5.0
25	4.0	4.4
26	3.6	3.9
27	3.2	3.5
28	2.8	3.1
29	2.5	2.7
30	2.2	2.4
31	2.0	2.2
32	1.8	2.0
33	1.7	1.8
34	1.5	1.6
35	1.4	1.4
36	1.3	1.3
37	1.2	1.1
38	1.1	1.0
39	1.0	0.9
40	0.9	0.8
41	0.8	0.7
42	0.8	0.7
43	0.7	0.6
44	0.7	0.5
45	0.5	0.5
46	0.5	0.4
47	0.5	0.4
48	0.4	0.3
49	0.4	0.3
50	0.4	0.3

Index of Multiple Correlation = 0.991

F-Ratio of Regression = 1824.5

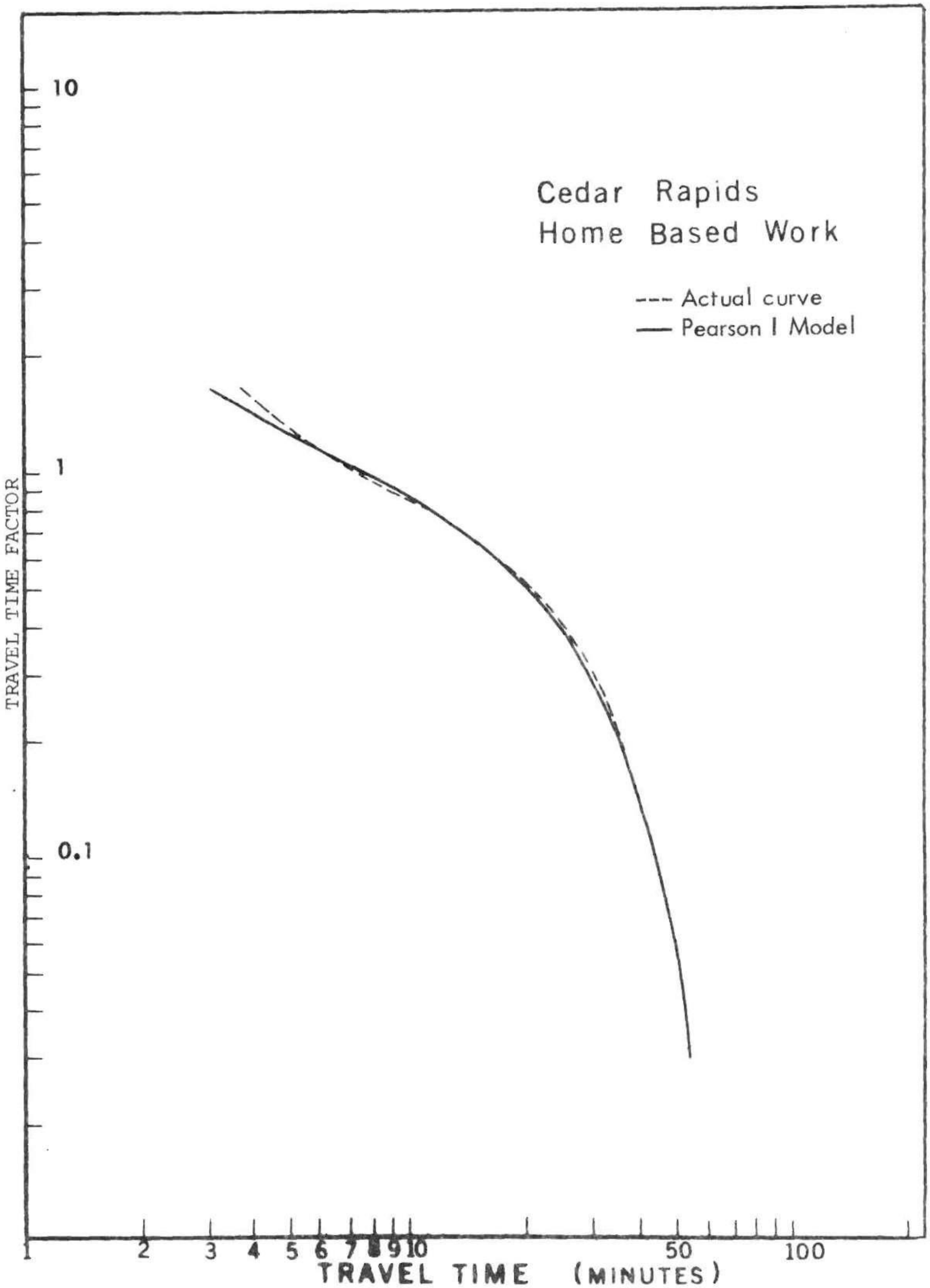


Figure 20. Pearson I Model for Cedar Rapids Home Based Work Travel Time Factor

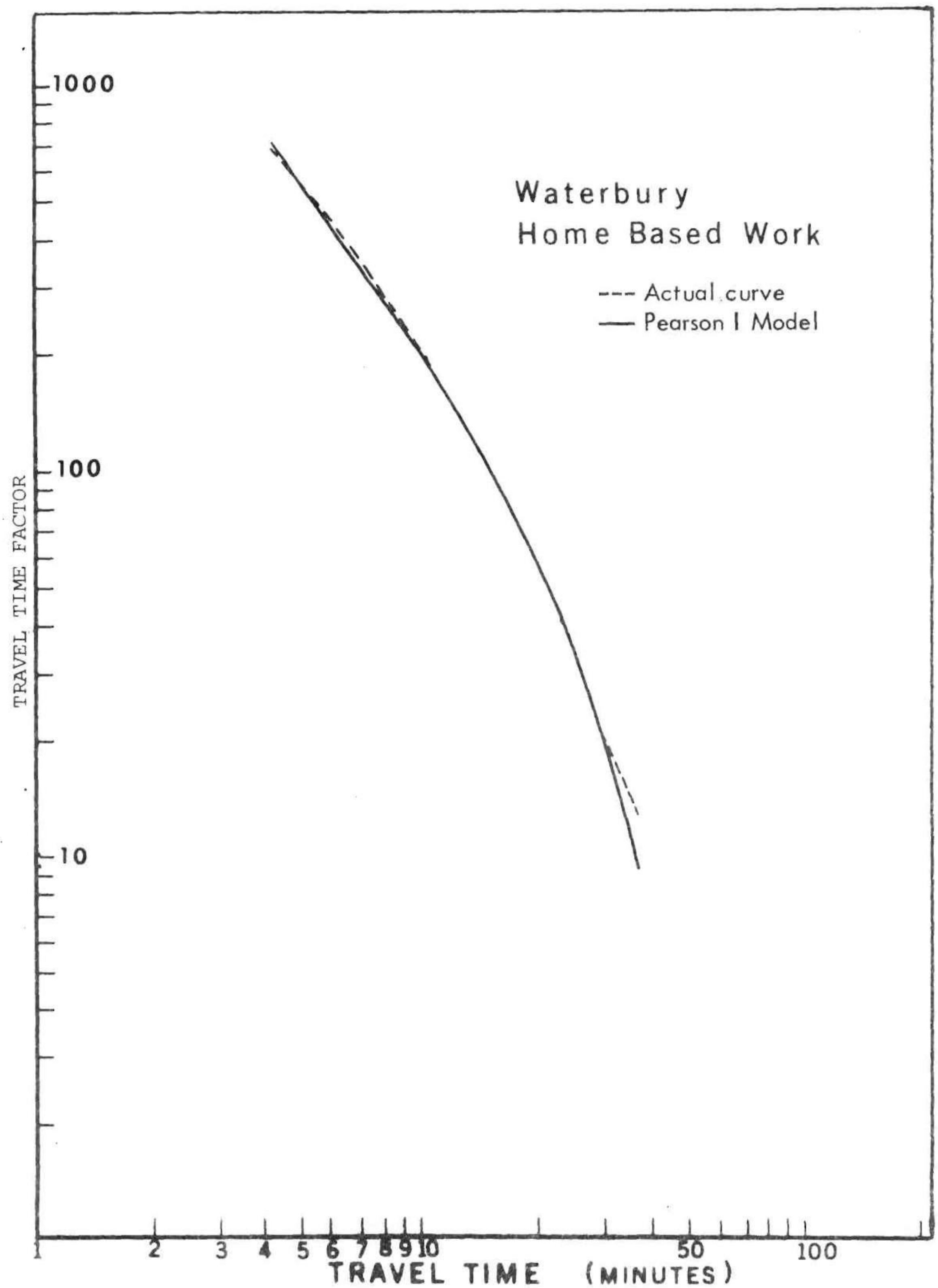


Figure 21. Pearson I Model for Waterbury Home Based Work Travel Time Factor

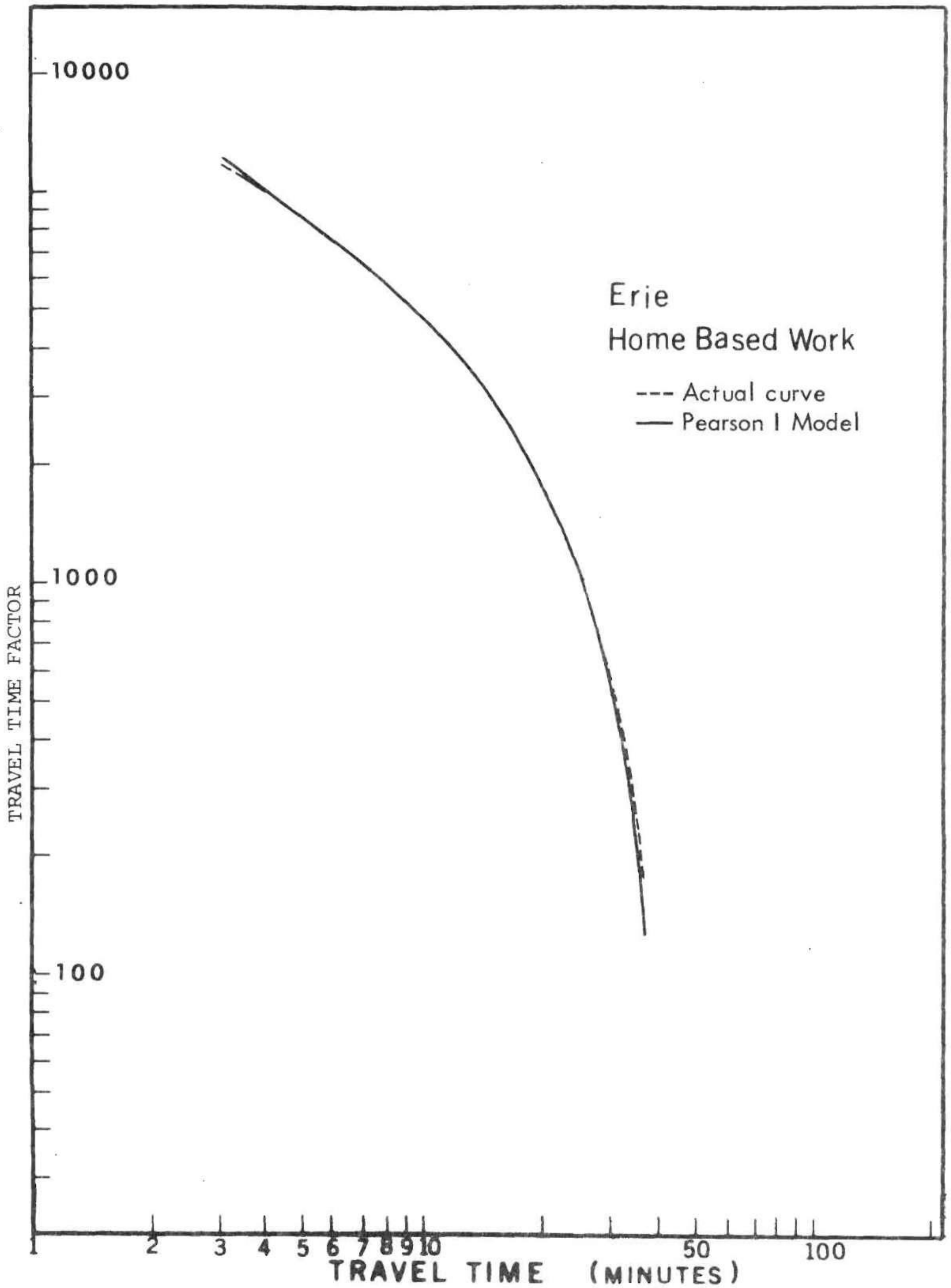


Figure 22. Pearson I Model for Erie Home Based Work Travel Time Factor

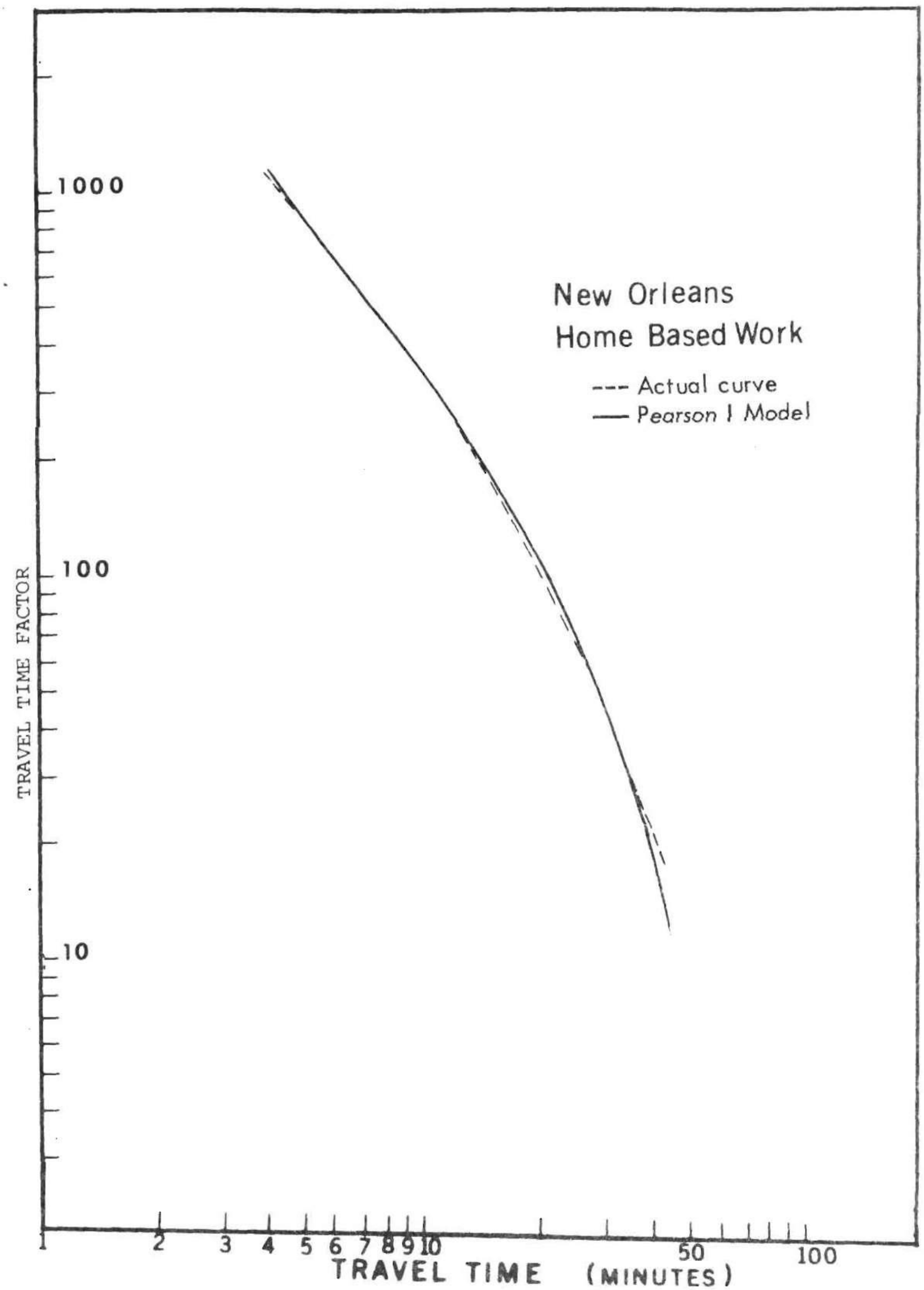


Figure 23. Pearson I Model for New Orleans Home Based Work Travel Time Factor

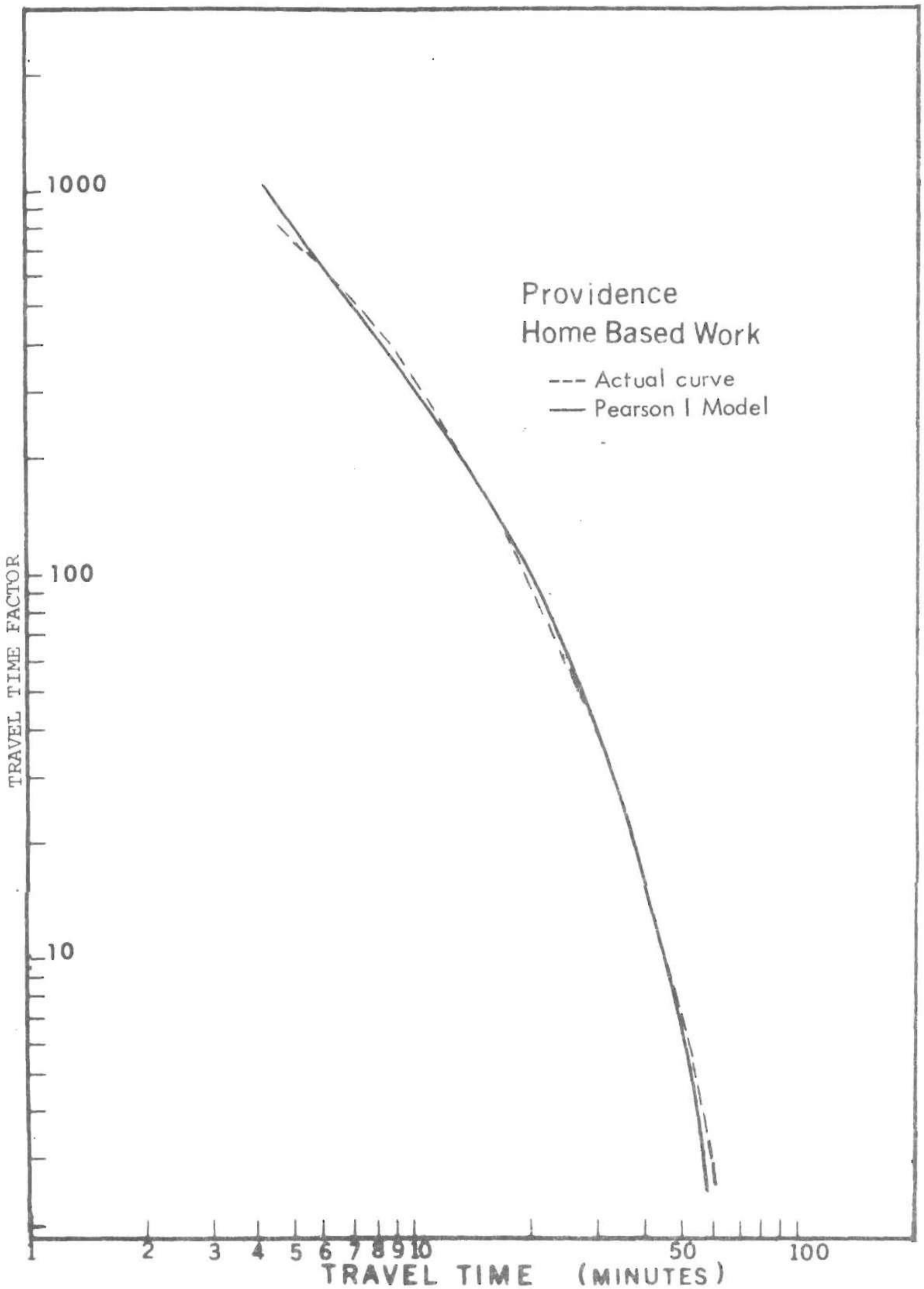


Figure 24. Pearson I Model for Providence Home Based Work Travel Time Factor

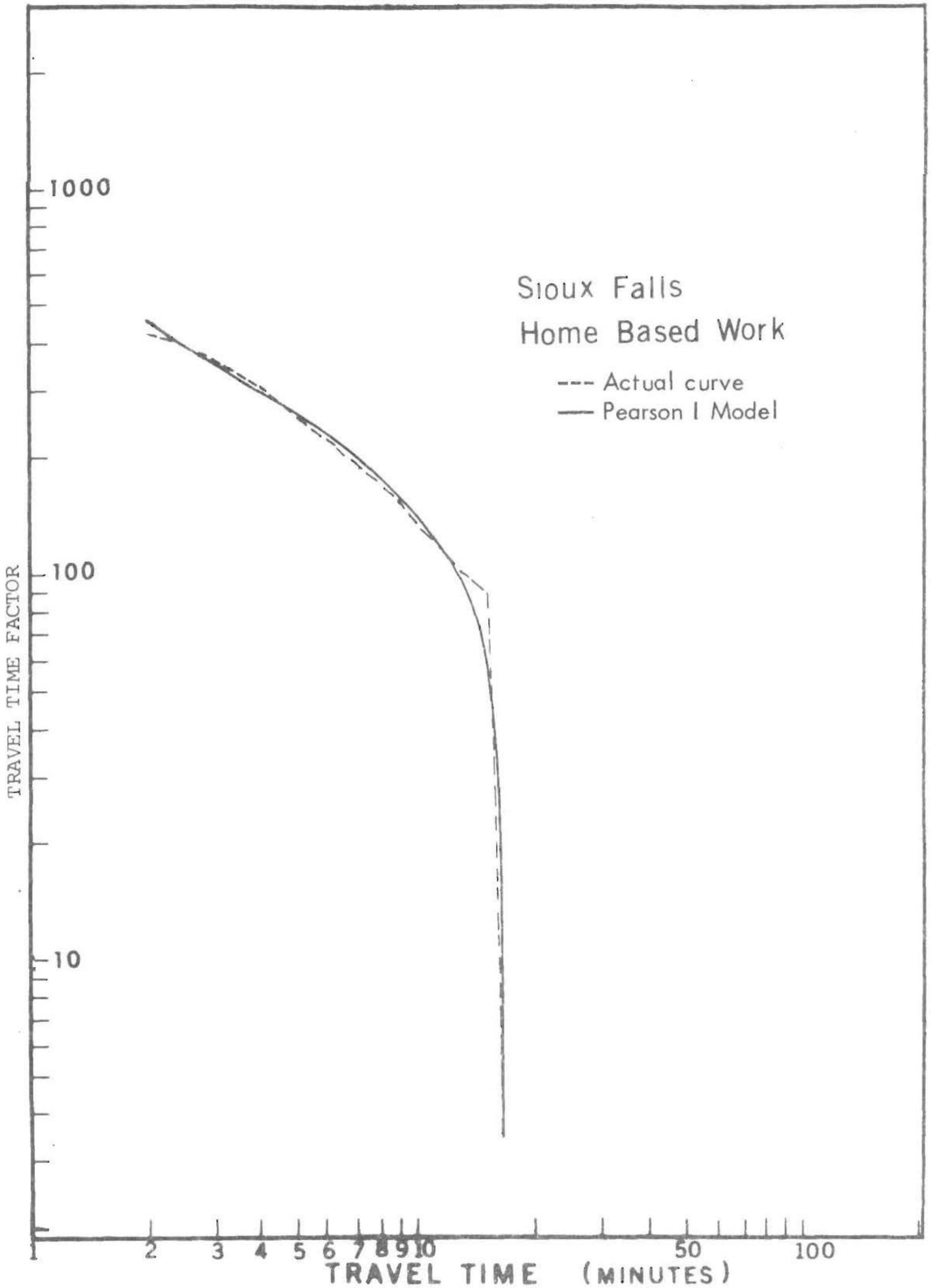


Figure 25. Pearson I Model for Sioux Falls Home Based Work Travel Time Factor

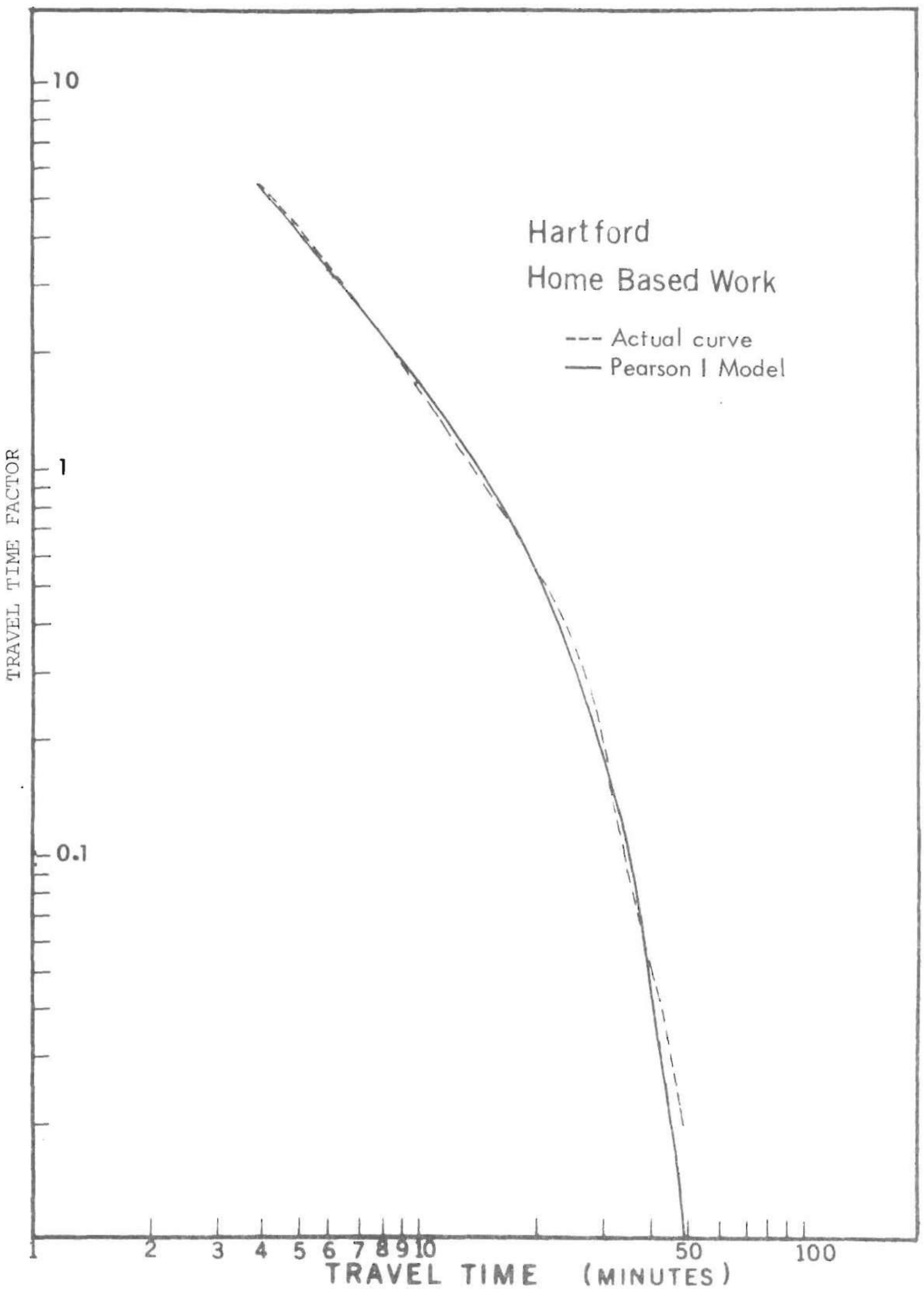


Figure 26. Pearson I Model for Hartford Home Based Work Travel Time Factor

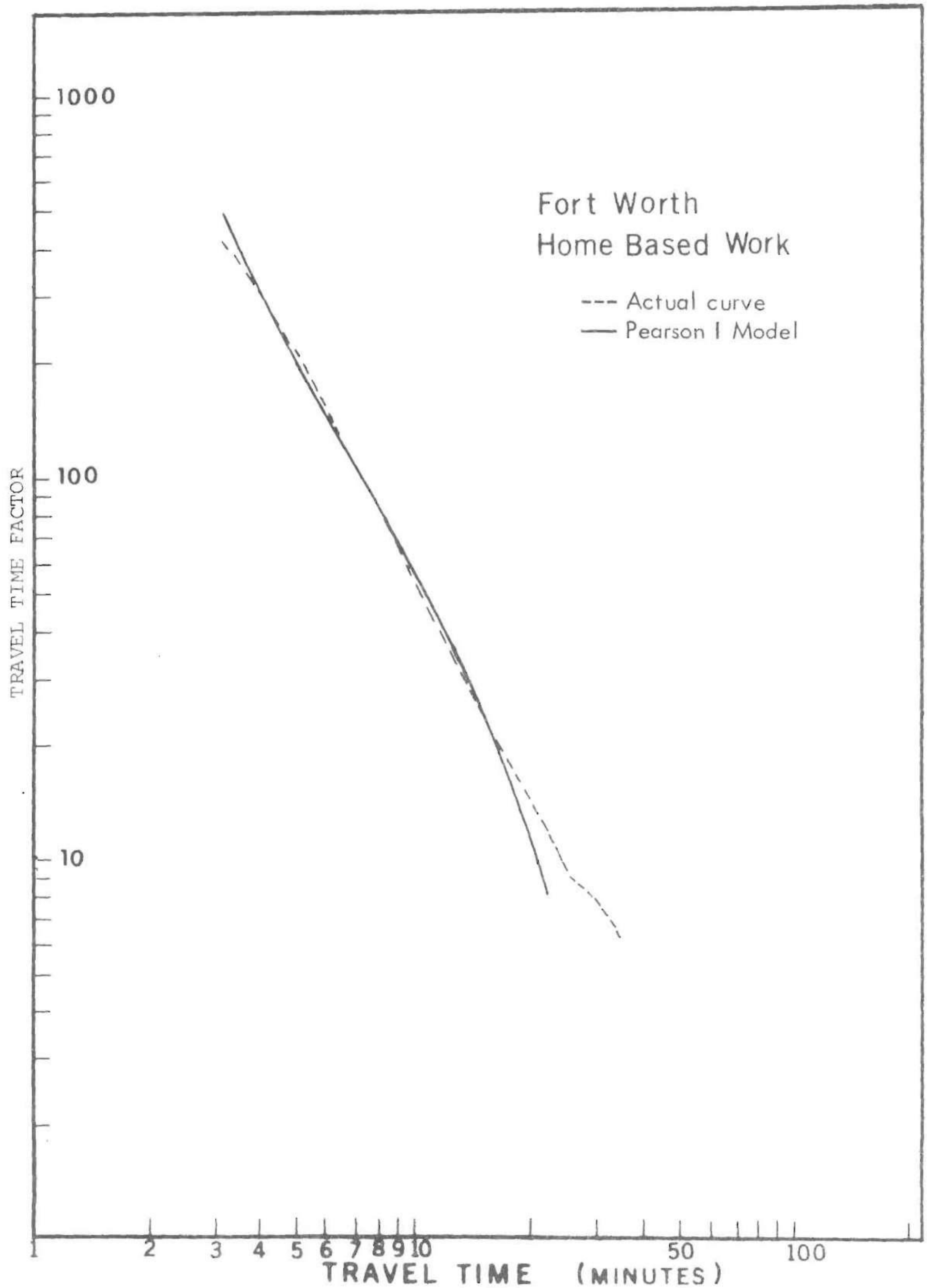


Figure 27. Pearson I Model for Fort Worth Home Based Work Travel Time Factor

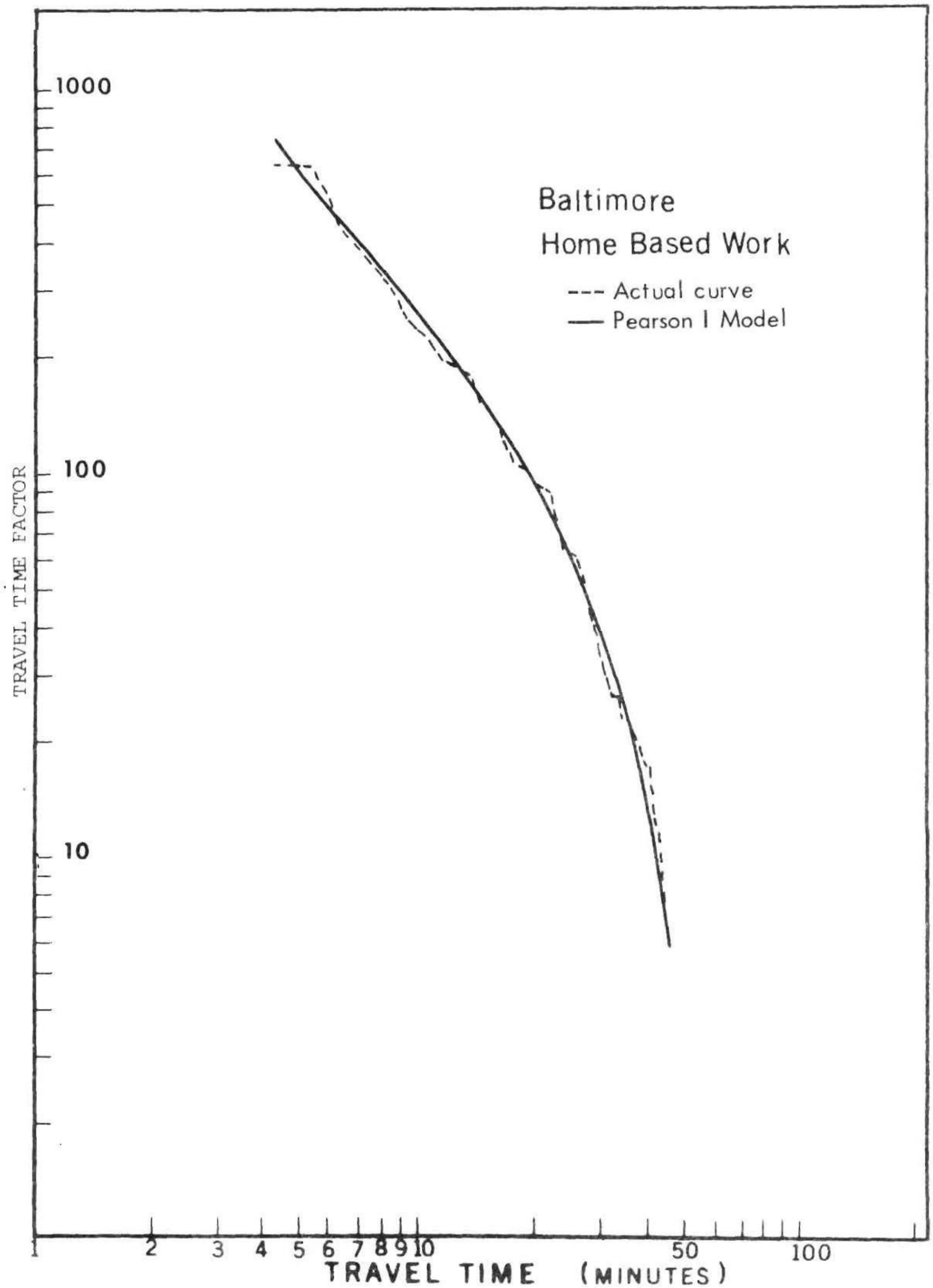


Figure 28. Pearson I Model for Baltimore
Home Based Work Travel Time Factor

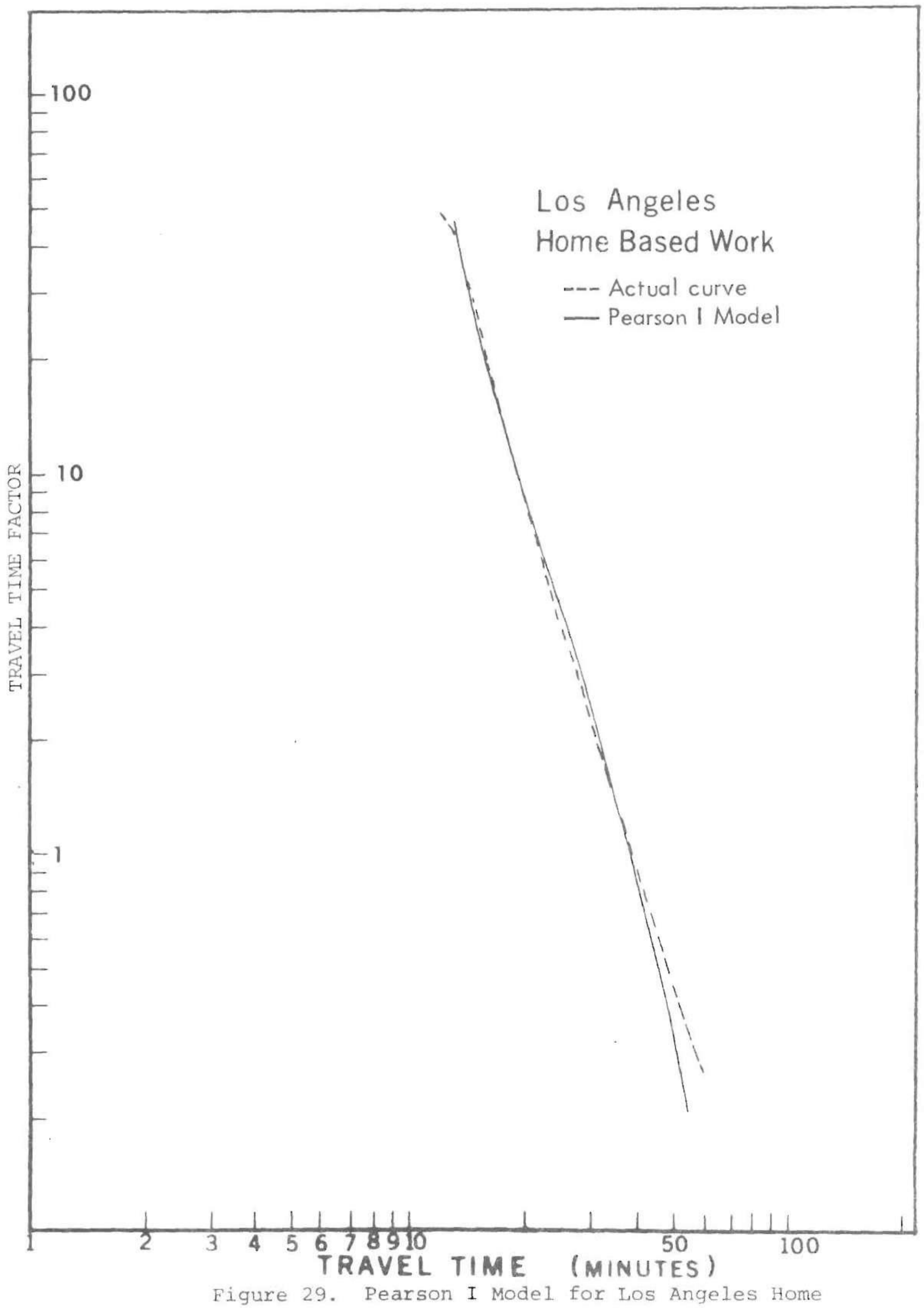


Figure 29. Pearson I Model for Los Angeles Home Based Work Travel Time Factor

Table 8. Linear Regression Analysis for m_1

Home Based Work

Model

$$m_1 = -.993 + .000933 \times (\text{Home Based Work Trips/1000 Population})$$

	Observed m_1	Predicted m_1
Cedar Rapids	-0.27	-0.21
Waterbury	-0.72	-0.62
Erie	-0.37	-0.59
New Orleans	-0.73	-0.73
Providence	-0.66	-0.63
Sioux Falls	-0.35	-0.53
Hartford	-0.53	-0.55
Fort Worth	-0.83	-0.64
Baltimore	-0.65	-0.72
Los Angeles	-0.76	-0.67

Standard Error of Estimate = 0.147
Correlation Coefficient:
Home Based Work Trips/1000 population r = 0.75

t - Value for Regression Coefficient:
t - Coefficient of Home Based Work Trips/1000 population t = 3.16
(Significant at 2 per cent)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-ratio	Level of Significance
Due to Regression	1	.196	.196	10.0	2%
About Regression	8	.157	.0196		
Total	9	.353			

Table 9. Linear Regression Analysis for m_2

Home Based Work

Model

$$\ln m_2 = 3.51 - 1.74 \ln \text{(Total Trips per Car)}$$

	Observed m_2	Predicted m_2
Cedar Rapids	1.48	1.02
Waterbury	4.16	4.64
Erie	1.89	2.48
New Orleans	2.84	3.78
Providence	5.40	4.74
Sioux Falls	0.48	0.46
Hartford	3.01	1.87
Fort Worth	3.89	2.66
Baltimore	2.19	4.22
Los Angeles	6.87	3.63

Standard Error of Estimate = 0.494 (Log Transformed)
Correlation Coefficient r = 0.79

t - value for Regression Coefficient = -3.63 (Significant at 1 per cent)

Analysis of Variance for Linear Regression

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	3.24	3.24	13.24	1%
About Regression	8	1.96	2.44		
Total	9	5.20			

Table 10. Regression Analysis for A

Home Based Work

Model

$$\ln A = -4.955 \times 10^{4} \times (\text{Home Based Work Trips})^{-1} + 4.52$$

	Observed A	Predicted A
Cedar Rapids	55.9	47.5
Waterbury	74.2	47.5
Erie	40.9	47.9
New Orleans	70.7	73.7
Providence	104.6	79.0
Sioux Falls	15.8	17.5
Hartford	60.6	79.0
Fort Worth	54.8	70.8
Baltimore	57.3	82.3
Los Angeles	128.3	90.0

Standard Error of Estimate 0.300 (Log Transformed)
Correlation Coefficient 0.866

t - value for Regression Coefficient:
t - Coefficient of Total Home Based Work Trips
$t = -4.90$ (Significant at 0.1 per cent)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	2.166	2.166	24.04	1%
About Regression	8	0.721	.090		
Total	9	2.887			

Table 11. Linear Regression Analysis for c

Home Based Work

Model

c = 2.63 - 0.0025 x (Home Based Work Trips per Thousand Population)

	Observed c	Predicted c
Cedar Rapids	0.7	0.5
Waterbury	1.7	1.6
Erie	1.2	1.5
New Orleans	1.9	1.9
Providence	2.2	1.7
Sioux Falls	1.0	1.4
Hartford	1.10	1.4
Fort Worth	2.0	1.7
Baltimore	1.9	1.9

Standard Error of Estimate = 0.36
Correlation Coefficient r = 0.77

t - value for Regression Coefficient:

t - value of Home Based Work Trips per Thousand Population
t = -3.23 (Significant at 2 per cent)

Analysis of Variance for Linear Regression:

Source of V Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	1.36	1.26	10.45	2%
About Regression	7	0.91	0.13		
Total	8	2.27			

APPENDIX B

NON HOME BASED MODELS

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

CEDAR RAPIDS--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	1.80	1.749
4	1.40	1.393
5	1.15	1.166
6	1.00	1.001
7	0.90	0.874
8	0.80	0.770
9	0.70	0.683
10	0.62	0.609
11	0.56	0.545
12	0.49	0.488
13	0.43	0.438
14	0.38	0.393
15	0.34	0.352
16	0.30	0.316
17	0.27	0.282
18	0.24	0.252
19	0.22	0.224
20	0.20	0.199
21	0.18	0.176
22	0.16	0.155
23	0.14	0.135
24	0.12	0.118
25	0.10	0.102
26	0.09	0.087
27	0.08	0.074
28	0.07	0.062
29	0.06	0.051
30	0.05	0.042
31	0.04	0.033
32	0.03	0.026
33	0.02	0.019
34	0.01	0.014
35	0.01	0.009

Index of Multiple Correlation = 0.999
F-Ratio of Regression = 17097.4

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

WATERBURY--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	775	820.3
5	665	663.7
6	560	540.6
7	460	441.6
8	370	360.9
9	300	294.7
10	237	240.1
11	187	195.1
12	155	158.0
13	117	127.4
14	98	102.2
15	81	81.6
16	65	64.8
17	55	51.1
18	42	40.1
19	35	31.1
20	28	24.0
21	22	18.4
22	17	13.9
23	12	10.4
24	10	7.7
25	8	5.6
26	6	4.1
27	5	2.9

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 4097.5

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

ERIE--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	3500	3796.5
4	2700	2657.3
5	2150	2041.2
6	1700	1640.6
7	1380	1354.0
8	1140	1136.5
9	960	964.8
10	800	825.5
11	700	710.2
12	600	613.2
13	510	530.8
14	450	460.1
15	390	399.0
16	340	345.9
17	295	299.5
18	250	259.0
19	220	223.5
20	200	192.3
21	175	165.0
22	150	140.9
23	130	119.8
24	110	101.4
25	100	85.2
26	87	71.1

Index of Multiple Correlation = 0.994
F-Ratio of Regression = 2135.3

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

PROVIDENCE--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	2230	2158.8
4	1500	1418.9
5	1080	1011.3
6	770	751.4
7	570	572.3
8	430	442.9
9	330	346.7
10	255	273.5
11	205	216.9
12	155	172.7
13	125	137.9
14	100	110.3
15	83	88.3
16	67	70.7
17	56	56.6
18	47	45.3
19	39	36.3
20	33	29.0

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 2380.0

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

SIOUX FALLS--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	120	121.2
5	100	98.6
6	80	82.8
7	70	70.9
8	60	61.3
9	55	53.2
10	44	46.3
11	38	40.1
12	32	34.4
13	30	29.1
14	26	24.0
15	23	19.0
16	14	13.7
17	5	7.6

Index of Multiple Correlation = 0.996
F-Ratio of Regression = 3133.4

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

HARTFORD--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	5.28	4.722
5	3.28	3.130
6	2.16	2.201
7	1.52	1.604
8	1.08	1.197
9	0.82	0.909
10	0.64	0.698
11	0.48	0.540
12	0.38	0.421
13	0.30	0.330
14	0.23	0.259
15	0.19	0.204
16	0.16	0.160
17	0.14	0.126
18	0.12	0.099
19	0.11	0.078
20	0.09	0.061
21	0.08	0.048
22	0.06	0.038
23	0.05	0.029
24	0.04	0.023
25	0.03	0.018
26	0.01	0.014
27	0.01	0.010
28	0.01	0.008

Index of Multiple Correlation = 0.990
F-Ratio of Regression = 790.9

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

FORT WORTH--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
4	226	229.7
5	172	164.7
6	133	125.6
7	100	99.2
8	77	80.0
9	62	65.5
10	50	54.1
11	41	44.9
12	35	37.5
13	30	31.3
14	26	26.2
15	23	21.9
16	20	18.3
17	17	15.3
18	15	12.7
19	13	10.5

Index of Multiple Correlation = 0.996
F-Ratio of Regression = 2560.3

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based

BALTIMORE--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
3	21660	20209
4	10920	11815
5	10404	8039
6	6583	5885
7	5100	4498
8	3885	3536
9	2760	2835
10	2116	2307
11	1650	1898
12	1458	1575
13	1161	1316
14	1002	1105
15	670	933
16	587	790
17	545	671
18	455	572
19	474	488
20	379	417
21	384	357
22	282	306
23	232	262
24	227	225
25	218	193
26	156	166
27	339	142
28	108	122
29	102	105
30	99	90
31	85	77
32	75	66
33	49	56
34	49	49

Index of Multiple Correlation = 0.985
F-Ratio of Regression = 2380.0

Table 12. Comparison of Travel Time Factors with Pearson I Models--Non Home Based LOS ANGELES--Non Home Based

Travel Time (Minutes)	Travel Time Factor	Pearson I Model
14	5.1	5.8
15	4.4	4.7
16	3.9	3.9
17	3.5	3.4
18	3.1	3.0
19	2.8	2.7
20	2.5	2.4
21	2.3	2.2
22	2.1	2.0
23	1.9	1.9
24	1.7	1.7
25	1.6	1.6
26	1.5	1.5
27	1.4	1.4
28	1.3	1.3
29	1.2	1.2
30	1.1	1.2
31	1.0	1.1
32	1.0	1.0
33	0.9	1.0
34	0.9	0.9
35	0.8	0.9
36	0.8	0.8
37	0.7	0.8
38	0.7	0.7
39	0.7	0.7
40	0.6	0.6
41	0.6	0.6
42	0.6	0.6
43	0.5	0.5
44	0.5	0.5
45	0.5	0.5
46	0.5	0.4
47	0.5	0.4
48	0.4	0.4
49	0.4	0.4
50	0.4	0.3

Index of Multiple Correlation = 0.987
F-Ratio of Regression = 2358.7

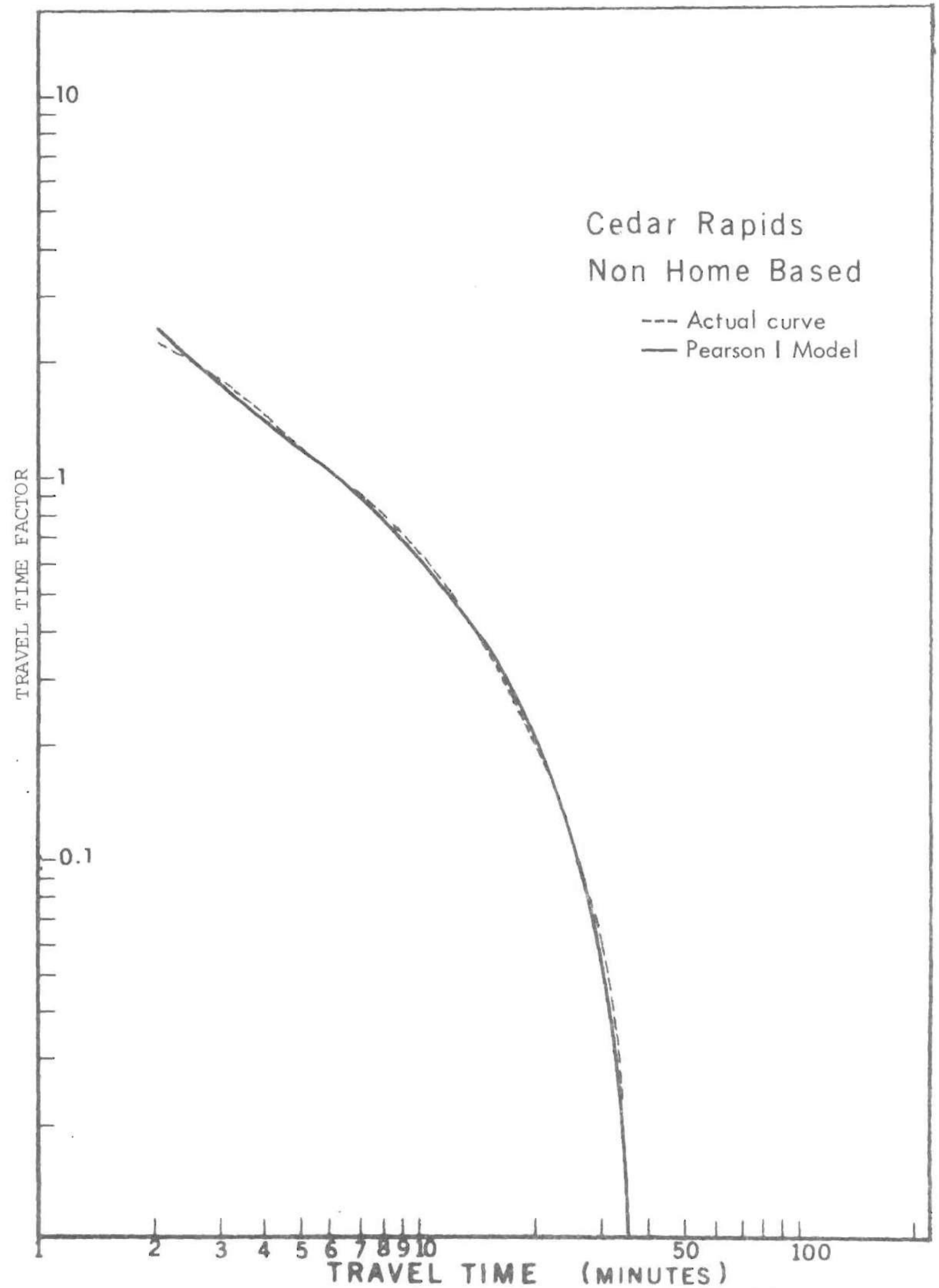


Figure 30. Pearson I Model for Cedar Rapids Non Home Based Travel Time Factor

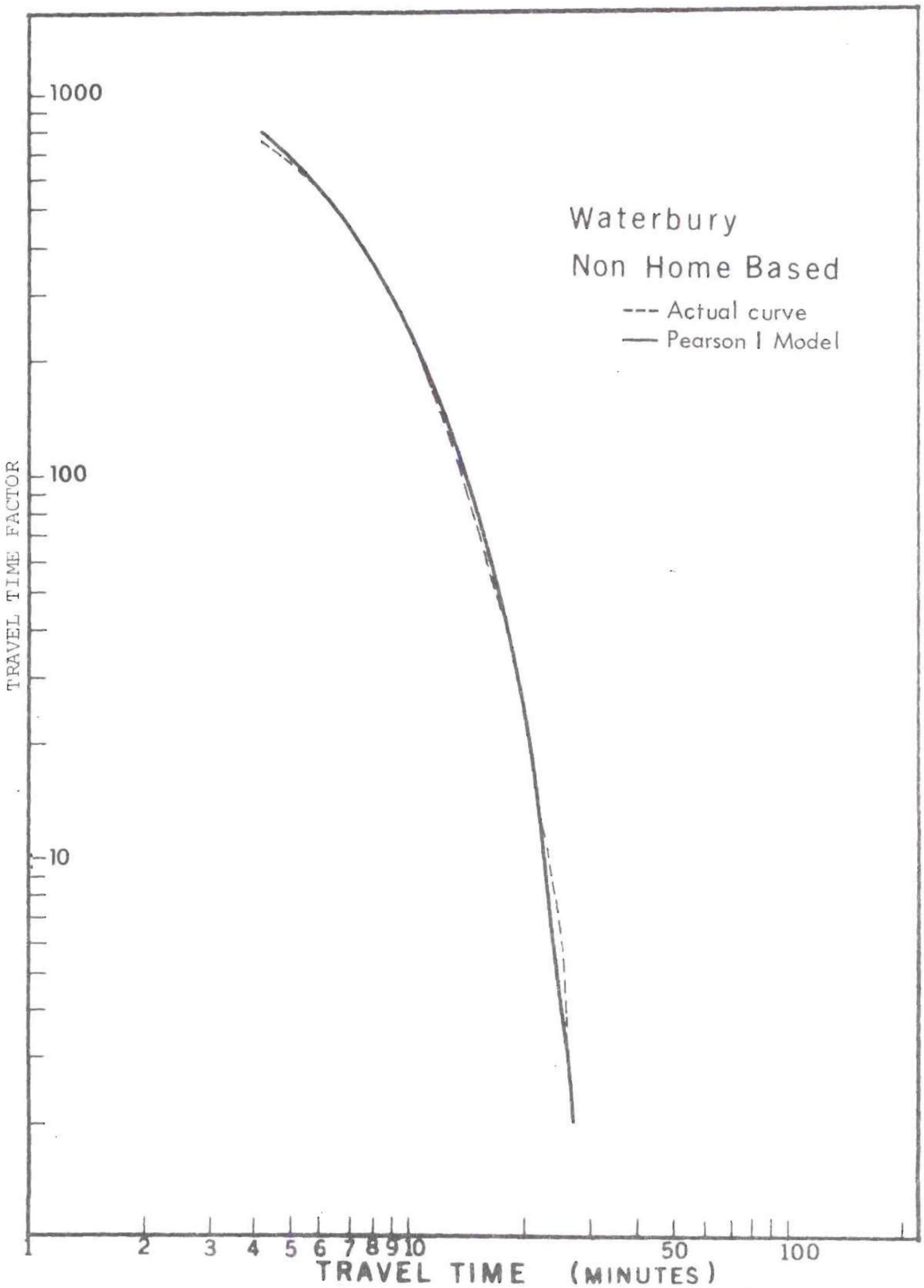


Figure 31. Pearson I Model for Waterbury Non Home Based Travel Time Factor

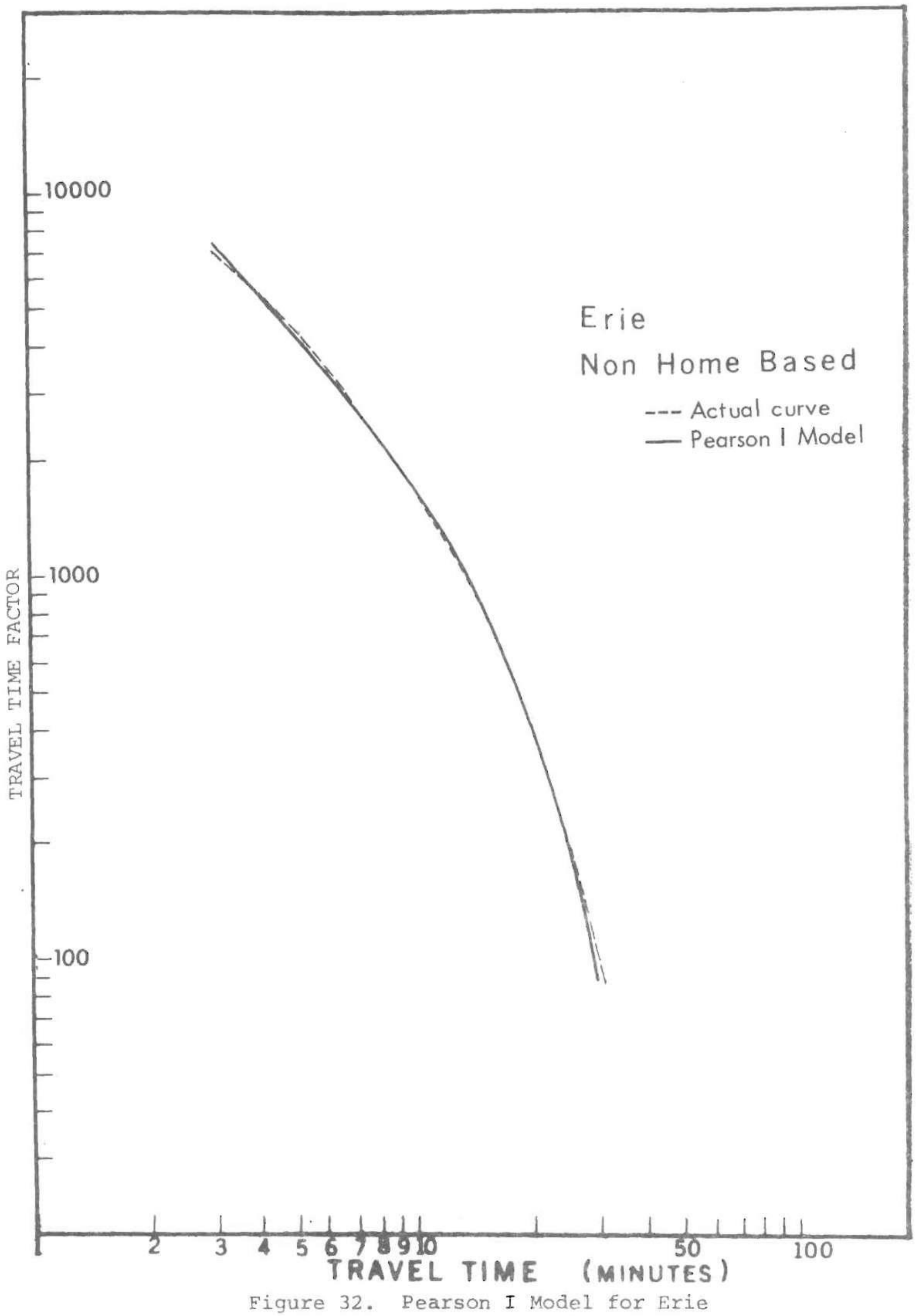


Figure 32. Pearson I Model for Erie
Non Home Based Travel Time Factor

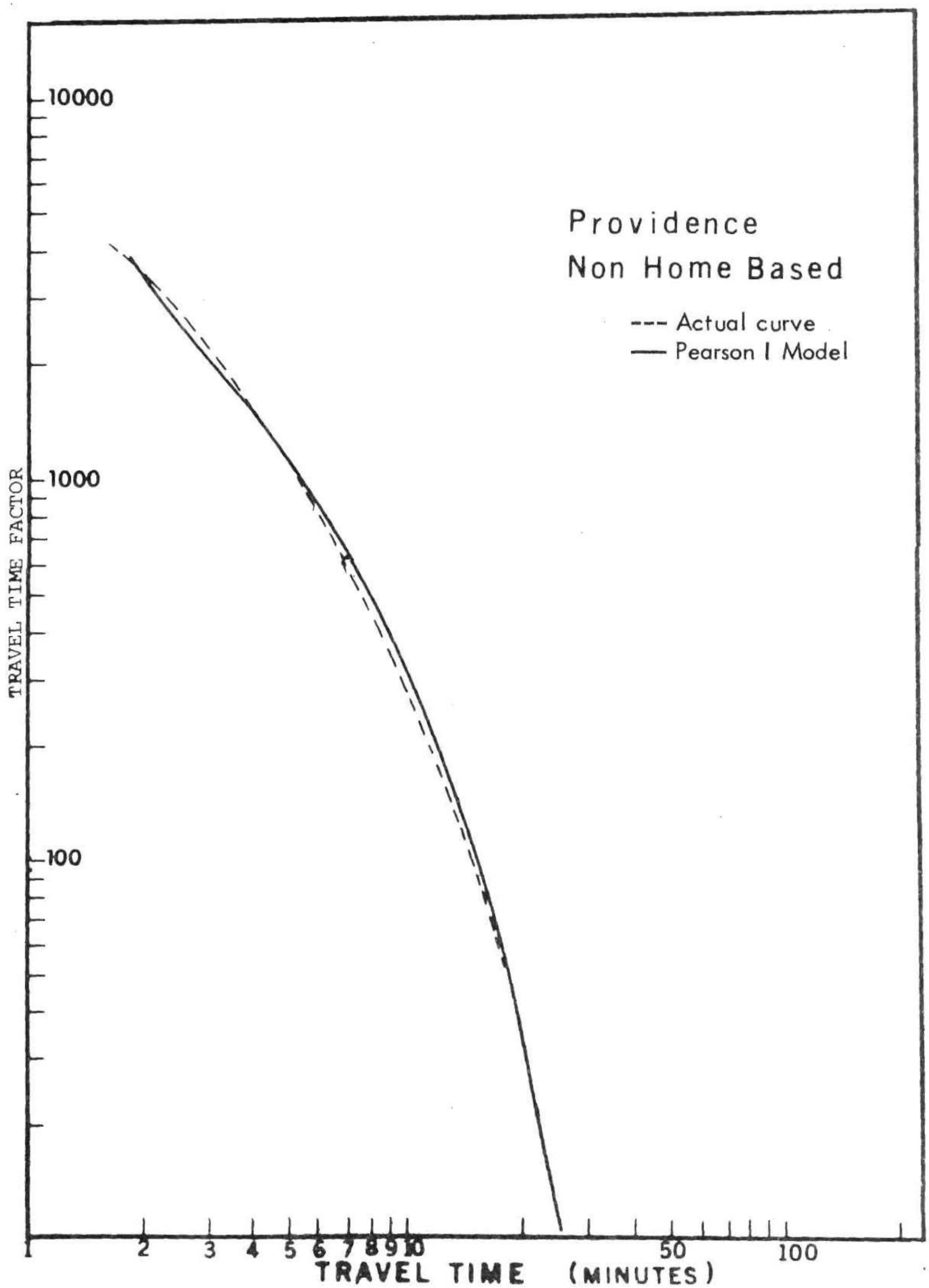


Figure 33. **Pearson I Model** for Providence Non Home **Based** Travel Time Factor

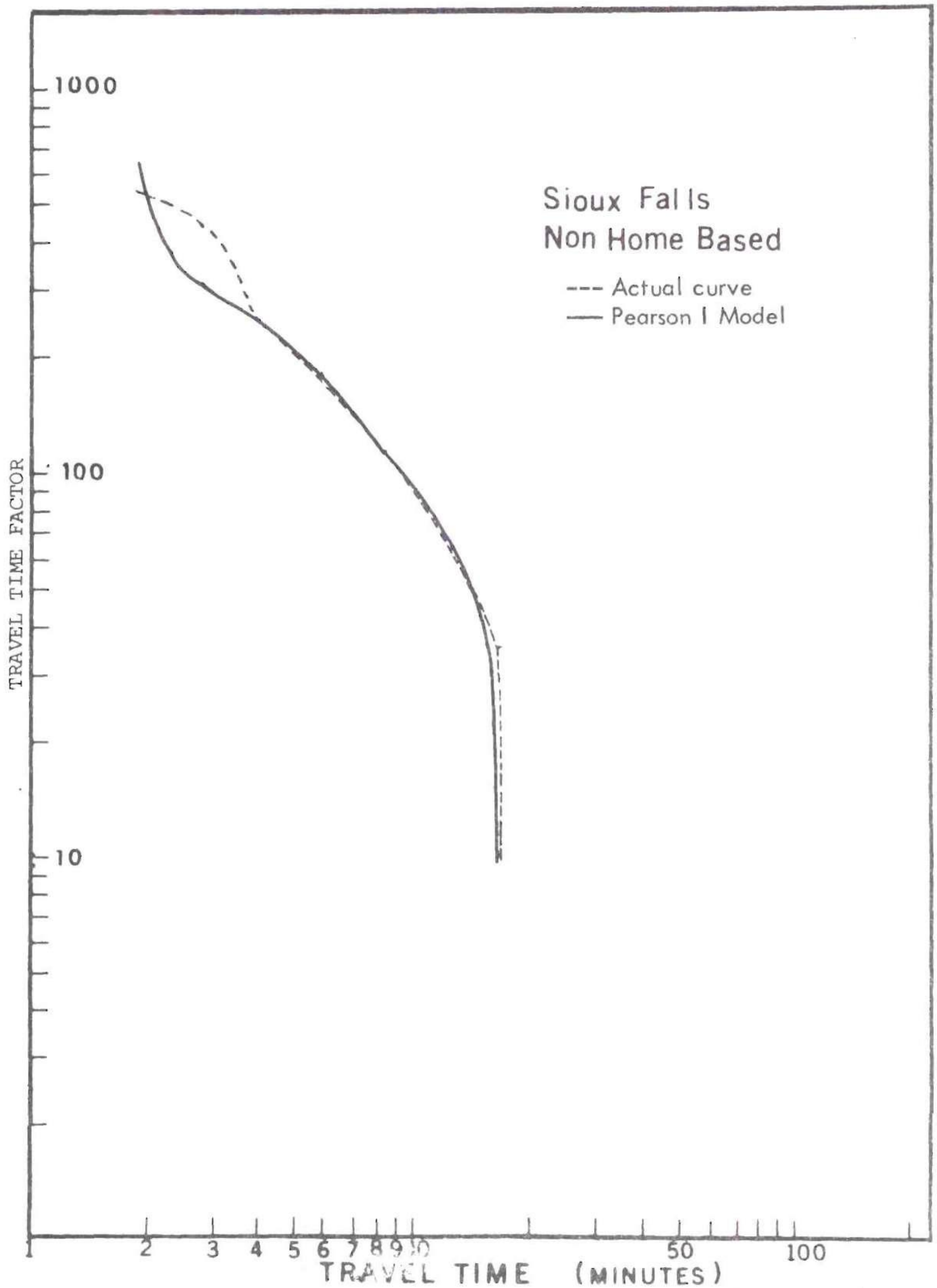


Figure 34. Pearson I Model for Sioux Falls Non Home Based Travel Time Factor

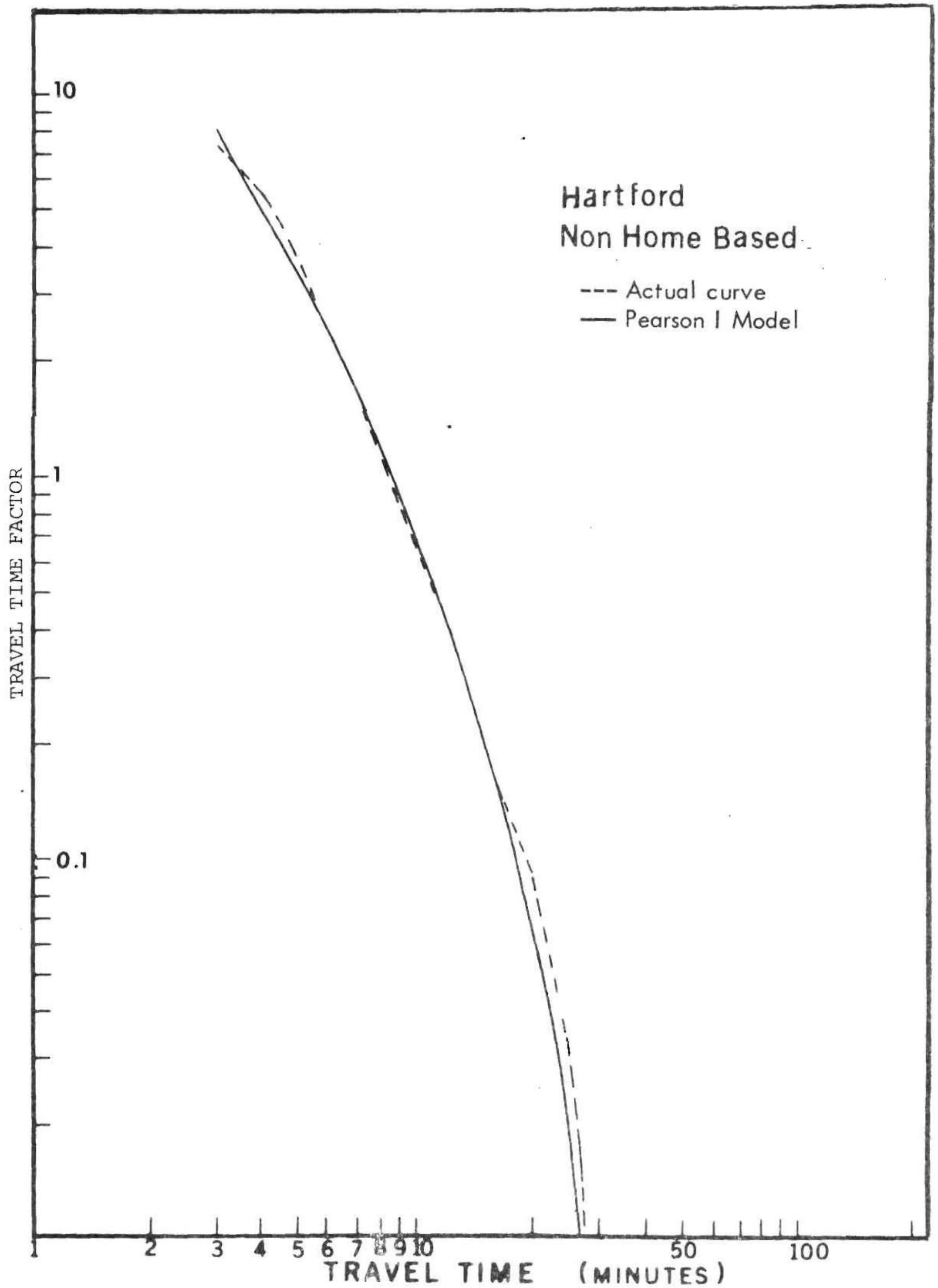


Figure 35. Pearson I Model for Hartford Non Home Based Travel Time Factor

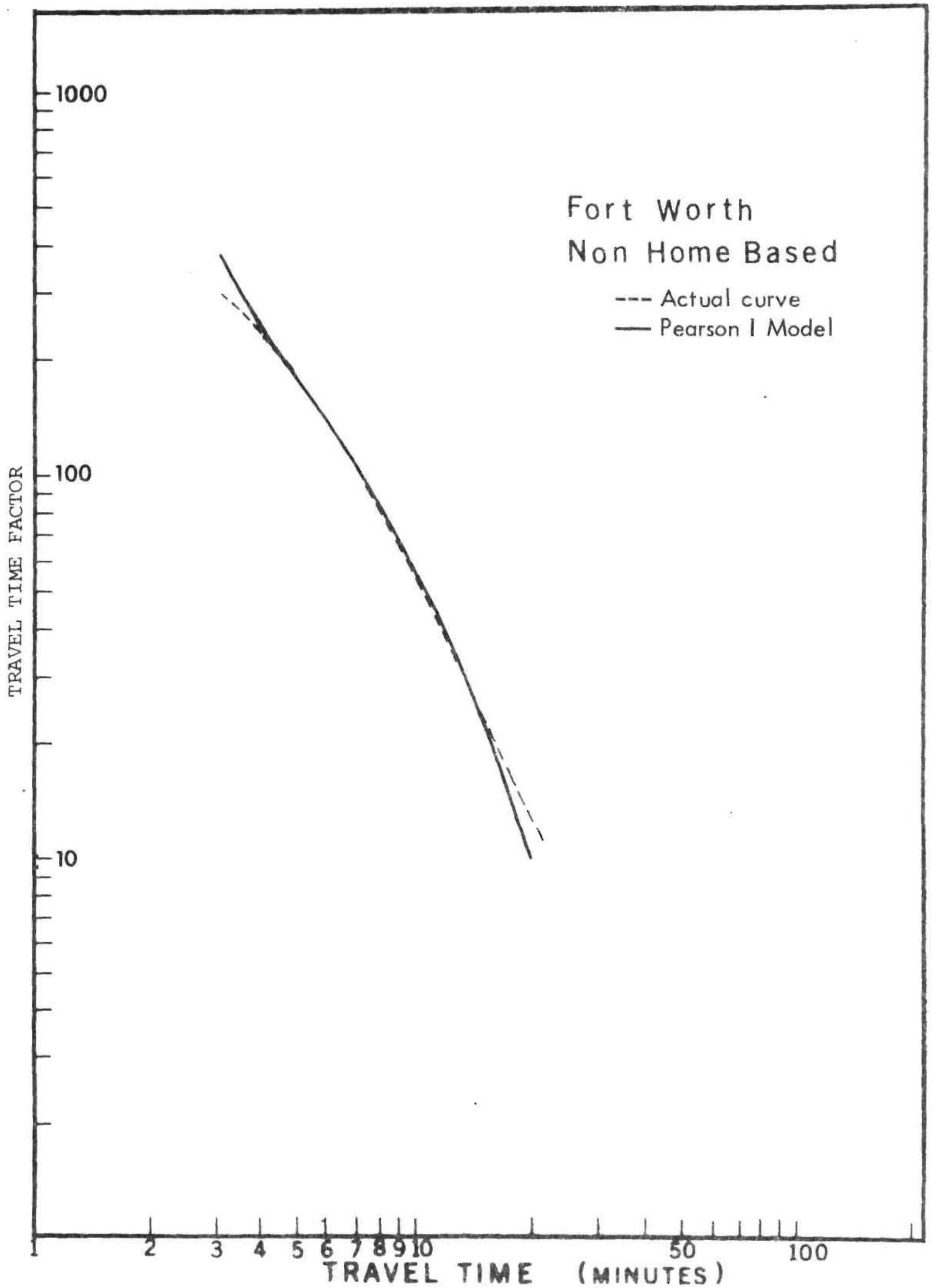


Figure 36. Pearson I Model for Fort Worth Non Home Based Travel Time Factor

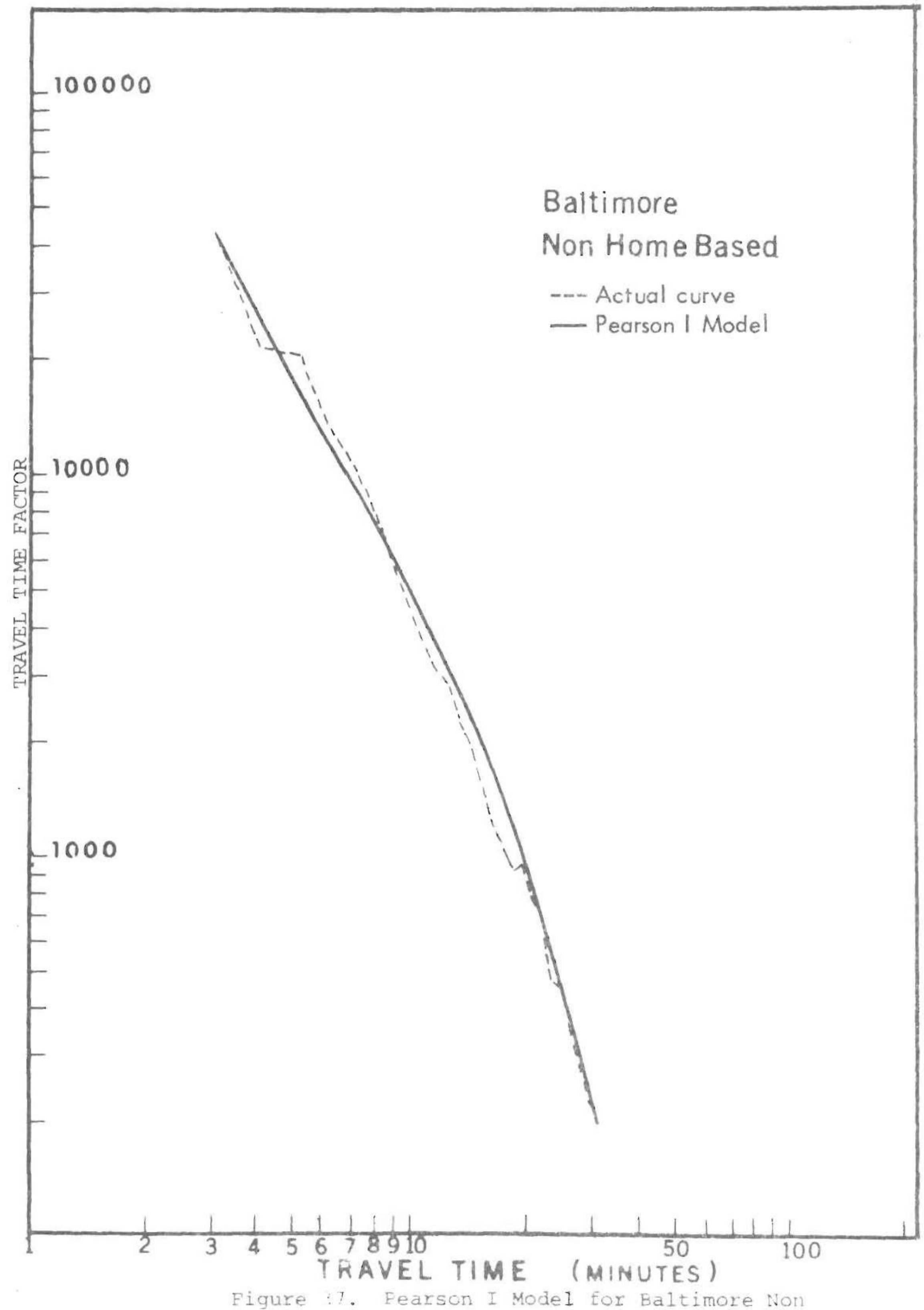


Figure 37. Pearson I Model for Baltimore Non Home Based Travel Time Factor

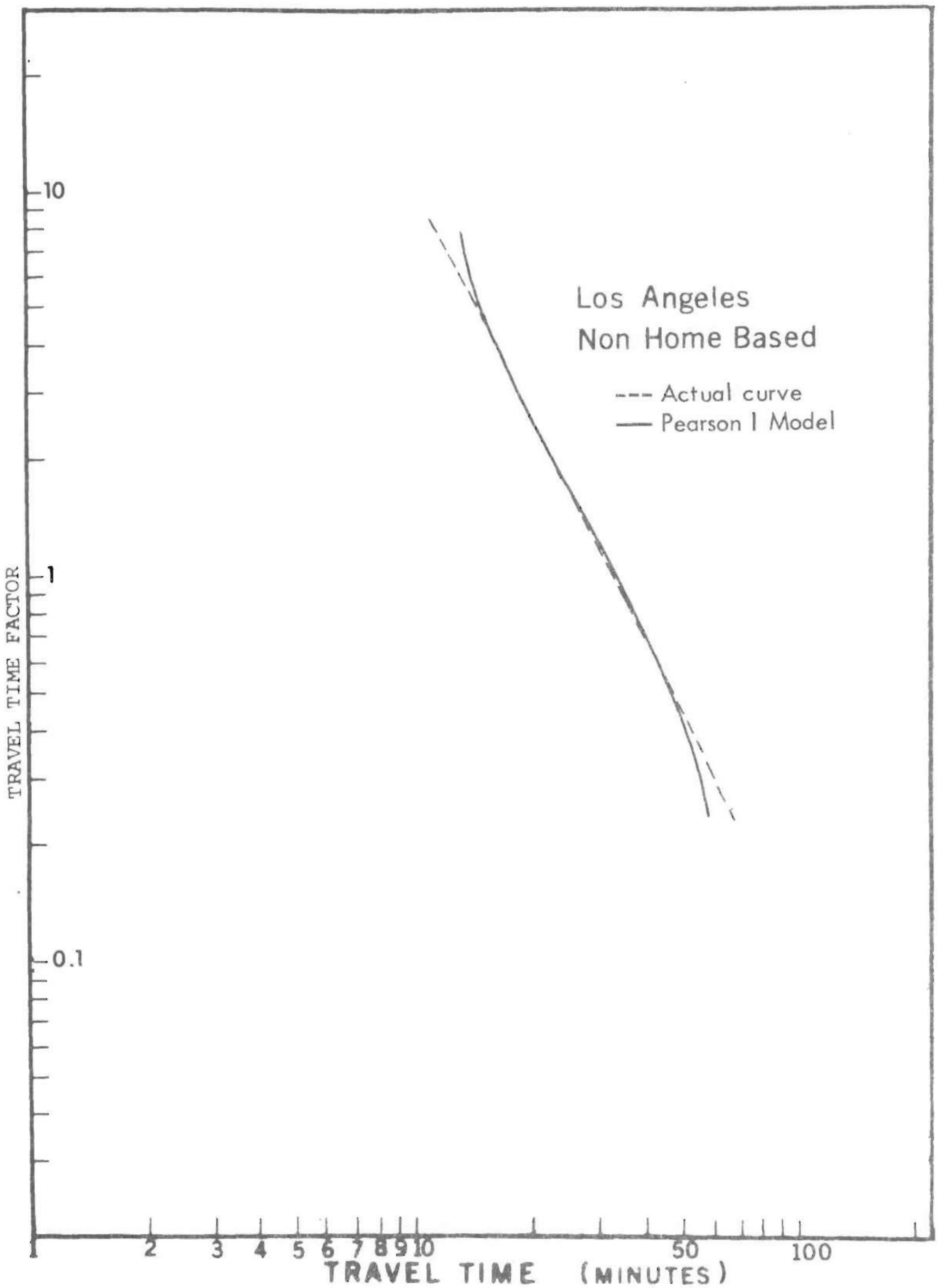


Figure 38. Pearson I Model for Los Angeles Non Home Based Travel Time Factor

Table 13. Multiple Regression Analysis for m_1

Non Home Based

Model

$$m_1 = 0.479 + 0.169 \times (\text{All Trips per Car}) - 1.56 \times (\text{Non Home Based Trips} \div \text{All Trips})$$

	Observed m_1	Predicted m_1
Cedar Rapids	-0.42	-0.54
Waterbury	-0.18	-0.14
Erie	-0.58	-0.64
Providence	-0.61	-0.58
Sioux Falls	-0.54	-0.57
Hartford	-0.91	-0.76
Fort Worth	-0.68	-0.67
Baltimore	-0.86	-0.70
Los Angeles	-0.64	-0.80

Standard Error of Estimate: 0.126
Partial Correlation Coefficients:
All Trips per Car $r = 0.82$
Non Home Based Trips = All Trips $r = 0.82$
Multiple Correlation Coefficient = 0.87

t - Values for Regression Coefficients:
All Trips per Car $t = 3.57$ (Significant at 2%)
Non Home Based Trips ÷ All Trips $t = 3.55$ (Significant at 2%)

Analysis of Variance for Multiple Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	2	0.297	0.148	9.23	2%
About Regression	6	0.096	0.016		
Total	8	0.393			

Table 14. Multiple Regression Analysis for m_2

Non Home Based

Model

$$m_2 = 6.56 + 5.86 \times 10^{-6} \times \text{Total Trips in Area} - 0.207 \times (\text{Non Home Based Trips} \div \text{Total Trips})$$

	Observed m_2	Predicted m_2
Cedar Rapids	1.95	3.48
Waterbury	7.02	5.23
Erie	2.89	3.85
Providence	11.48	9.46
Sioux Falls	0.62	0.79
Hartford	8.05	7.13
Fort Worth	3.11	6.27
Baltimore	9.92	10.40

Standard Error of Estimate = 2.18

Partial Correlation Coefficient
All Trips r = 0.867
Non Home Based Trips ÷ All Trips = -0.78
Multiple Correlation Coefficient = 0.89

t - Values for Regression Coefficients
All Trips t = 3.89 (Significant at 2% level)
Non Home Based Trips ÷ Total Trips: t = -2.75 (Significant at 5%)

Analysis of Variance for Multiple Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Regression on All Trips	53.52	1	53.52		
Addition of NHB%	35.91	1	35.91	7.57	5%
Joint Regression	89.43	2	44.43	9.42	2%
Residual	23.71	5	4.74		
Total	113.14	7			

Table 15. Linear Regression Analysis for m_2

Non Home Based

Model

$$m_2 = 2.05 + 0.0000211 \times \text{(Number of Cars in Study Area)}$$

	Observed m_2	Predicted m_2
Cedar Rapids	1.95	2.67
Waterbury	7.02	3.39
Erie	2.89	3.21
Providence	11.48	7.90
Sioux Falls	0.62	2.40
Hartford	8.05	8.31
Fort Worth	3.11	5.82
Baltimore	9.92	11.27

Standard Error of Estimate = 2.56
Correlation Coefficient = 0.81

t - Value for Regression Coefficient:

Number of Cars in Study Area t = 3.34 (Significant at 2%)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	73.65	73.65	11.19	2%
About Regression	6	39.50	6.58		
Total	7	113.15			

Table 16. Regression Analysis for A

Non Home Based

Model

$\log A = 6.55 - 0.417 \times \ln$ (Non Home Based Trips ÷ Study Area in Sq.Mi.)

$$A = e^{6.55} \times (\text{Non Home Based Trips} \div \text{Study Area in Sq.Mi.})^{-0.417}$$

	Observed ln (A)	Predicted ln (A)
Cedar Rapids	3.84	4.05
Waterbury	3.65	3.54
Erie	3.75	3.72
Providence	4.41	4.77
Sioux Falls	2.80	3.23
Hartford	4.22	3.91
Fort Worth	3.87	4.01
Baltimore	4.69	4.08
Los Angeles	4.05	4.00

Standard Error of Estimate = 0.348 (log transformed)
Correlation Coefficient = 0.793

t - Value for Regression Coefficient:
Non Home Based Trips ÷ Study Area t = -3.45 (Significant at 1%)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	1.441	1.441	11.9	2%
About Regression	7	0.849	0.121		
Total	8	2.290			

Table 17. Linear Regression Analysis for c

Non Home Based

Model

$$c = 1.51 - 0.17 \times \text{(Non Home Based Trips per Car)}$$

	Observed c	Predicted c
Cedar Rapids	1.02	1.24
Waterbury	0.90	0.98
Erie	1.42	1.36
Providence	1.20	1.45
Sioux Falls	1.10	0.98
Hartford	1.07	1.16
Fort Worth	1.68	1.34
Baltimore	1.52	1.39

Standard Error of Estimate 0.2
Correlation Coefficient r = 0.71

t - Value of Regression Coefficient:
Non Home Based Trips per Car = -2.23 (Significant at 7% level)

Analysis of Variance for Multiple Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	0.235	0.235	4.98	7%
About Regression	6	0.283	0.047		
Total	7	0.518			

APPENDIX C

SHOPPING MODELS

Table 18. Comparison of Travel Time Factor with Pearson III Model--Shopping

WATERBURY--Shopping

Travel Time (Minutes)	Travel Time Factor	Pearson III Model
2	1400	1418.7
3	900	833.2
4	540	534.3
5	320	355.8
6	240	242.1
7	175	167.0
8	120	116.2
9	85	81.5
10	58	40.6
11	40	28.9
12	27	20.6
13	18	14.7
14	12	10.5
15	9	10.5
16	7	7.5
17	6	5.4
18	5	3.9

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 2913.1

Table 18. Comparison of Travel Time Factor with Person III Model--Shopping (Continued)

ERIE--Shopping

Travel Time (Minutes)	Travel Time Factor	Pearson III Model
2	6000	5485
3	3600	3271
4	2300	2175
5	1550	1517
6	1050	1086
7	720	792
8	530	584
9	410	434
10	310	325
11	225	245
12	170	185
13	130	141
14	100	107
15	80	82
16	67	63
17	53	48
18	41	37

Index of Multiple Correlation = 0.991
F-Ratio of Regression = 823.7

Table 18. Comparison of Travel Time Factor with Pearson III Model--Shopping (Continued)

PROVIDENCE--Shopping

Travel Time (Minutes)	Travel Time Factor	Pearson III Model
2	3600	3750
3	2400	2220
4	1500	1494
5	1050	1050
6	770	770
7	550	573
8	420	432
9	300	251
10	235	251
11	190	195
12	145	151
13	120	118
14	93	92
15	75	72
16	61	56
17	52	44

Index of Multiple Correlation = 0.996
F-Ratio of Regression = 2225.8

Table 18. Comparison of Travel Time Factor with Pearson III Model--Shopping (Continued)

FORT WORTH--Shopping

Travel Time (Minutes)	Travel Time Factor	Pearson III Model
2	2860	2998.3
3	1500	1573.4
4	890	905.3
5	553	542.6
6	307	332.8
7	175	207.2
8	107	130.3
9	72	82.6
10	50	52.7
11	37	33.7
12	26	21.7
13	20	14.0
14	15	9.1
15	12	5.7
16	10	3.8

Index of Multiple Correlation = 0.997
F-Ratio of Regression = 1951.2

Table 18. Comparison of Travel Time Factor with Pearson III Model--Shopping (Continued)

HARTFORD--Shopping

Travel Time (Minutes)	Travel Time Factor	Pearson III Model
3	7.36	8.00
4	5.28	4.87
5	3.28	3.24
6	2.16	2.26
7	1.52	1.63
8	1.08	1.20
9	0.82	0.90
10	0.64	0.68
11	0.48	0.51
12	0.38	0.40
13	0.30	0.31
14	0.25	0.24
15	0.21	0.19
16	0.18	0.15

Index of Multiple Correlation = 0.990
F-Ratio of Regression = 760.4

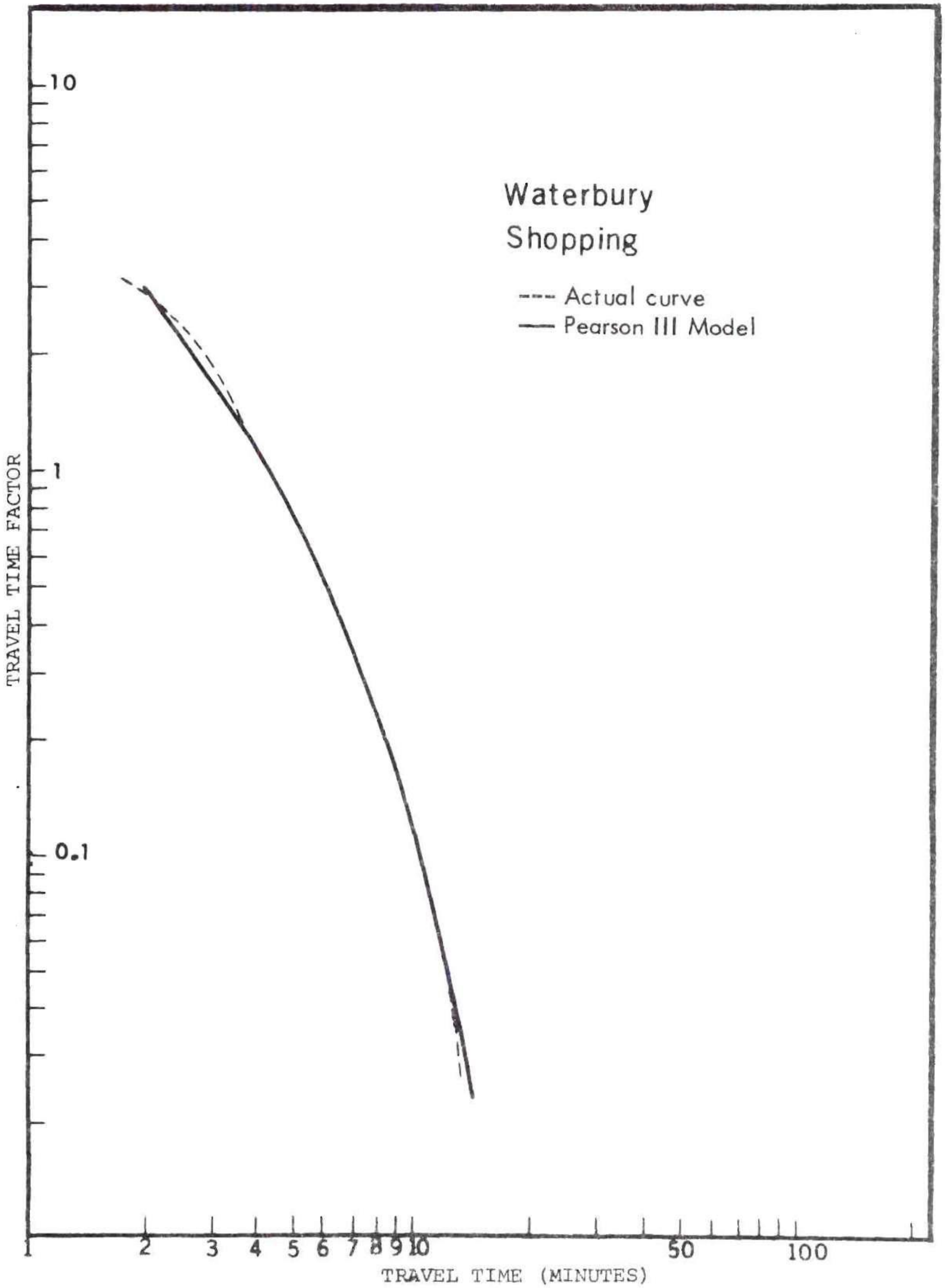


Figure 39. Pearson III Model for Waterbury Shopping Travel Time Factor

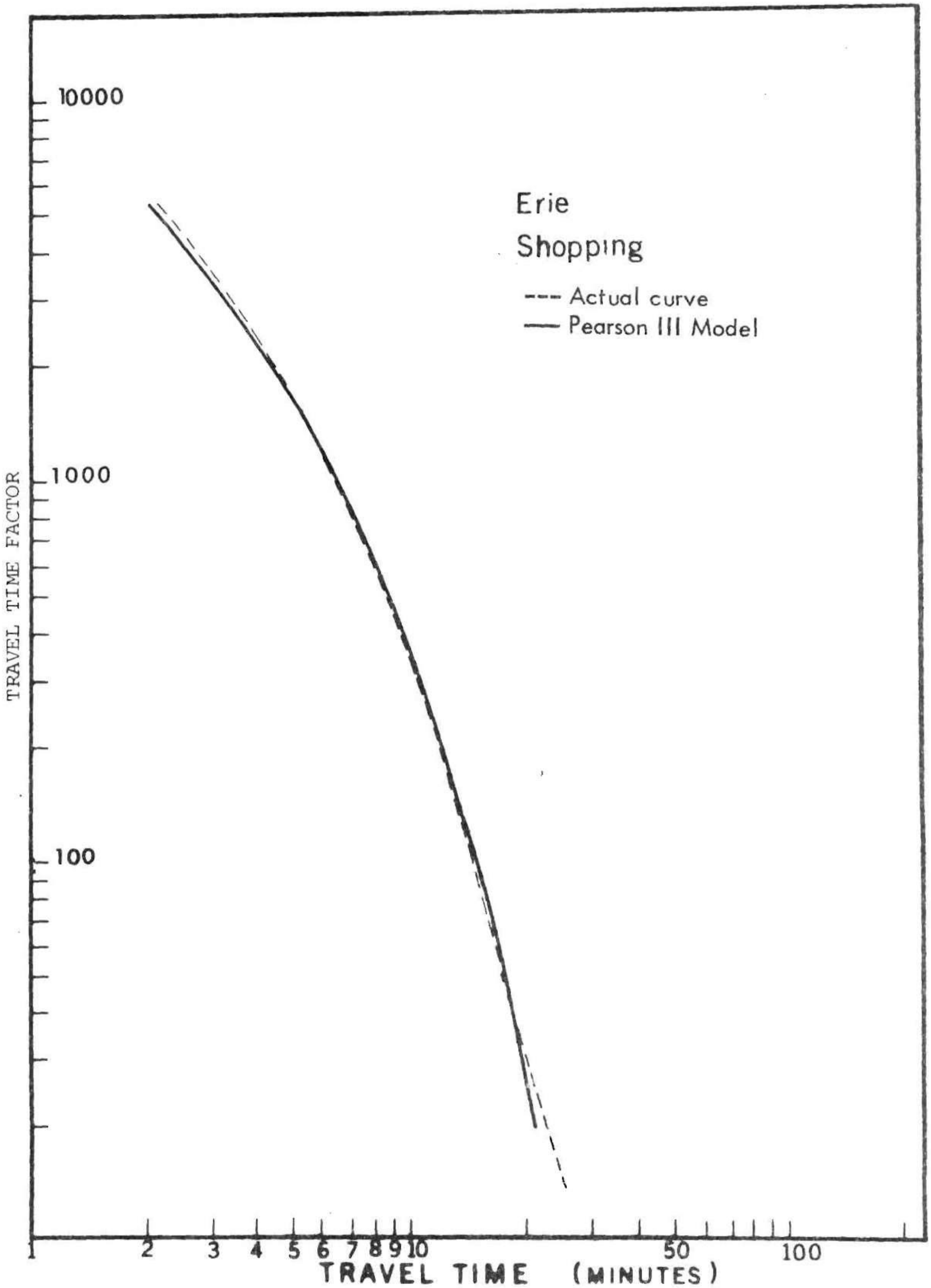


Figure 40. Pearson III Model for Erie Shopping Travel Time Factor

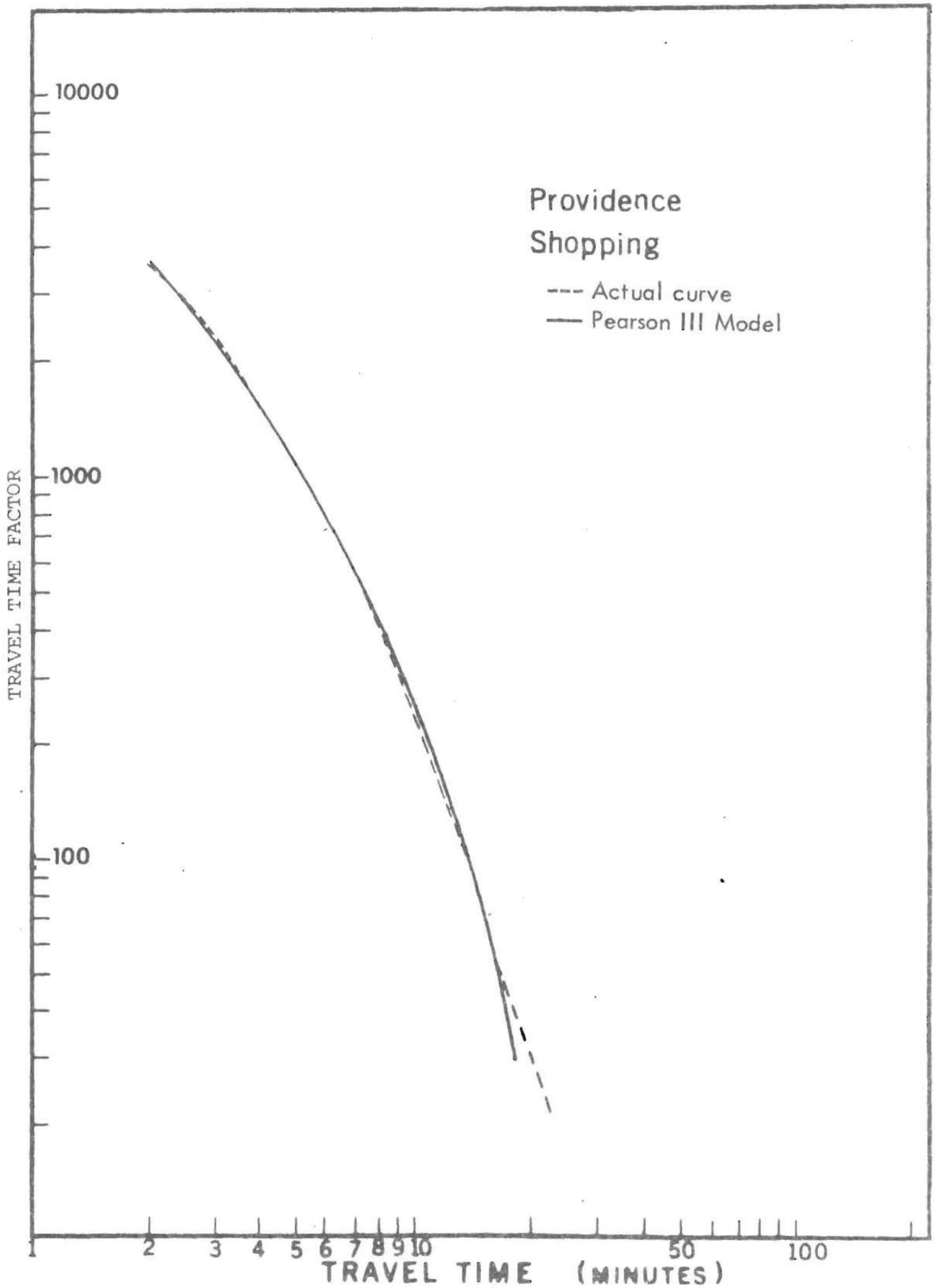
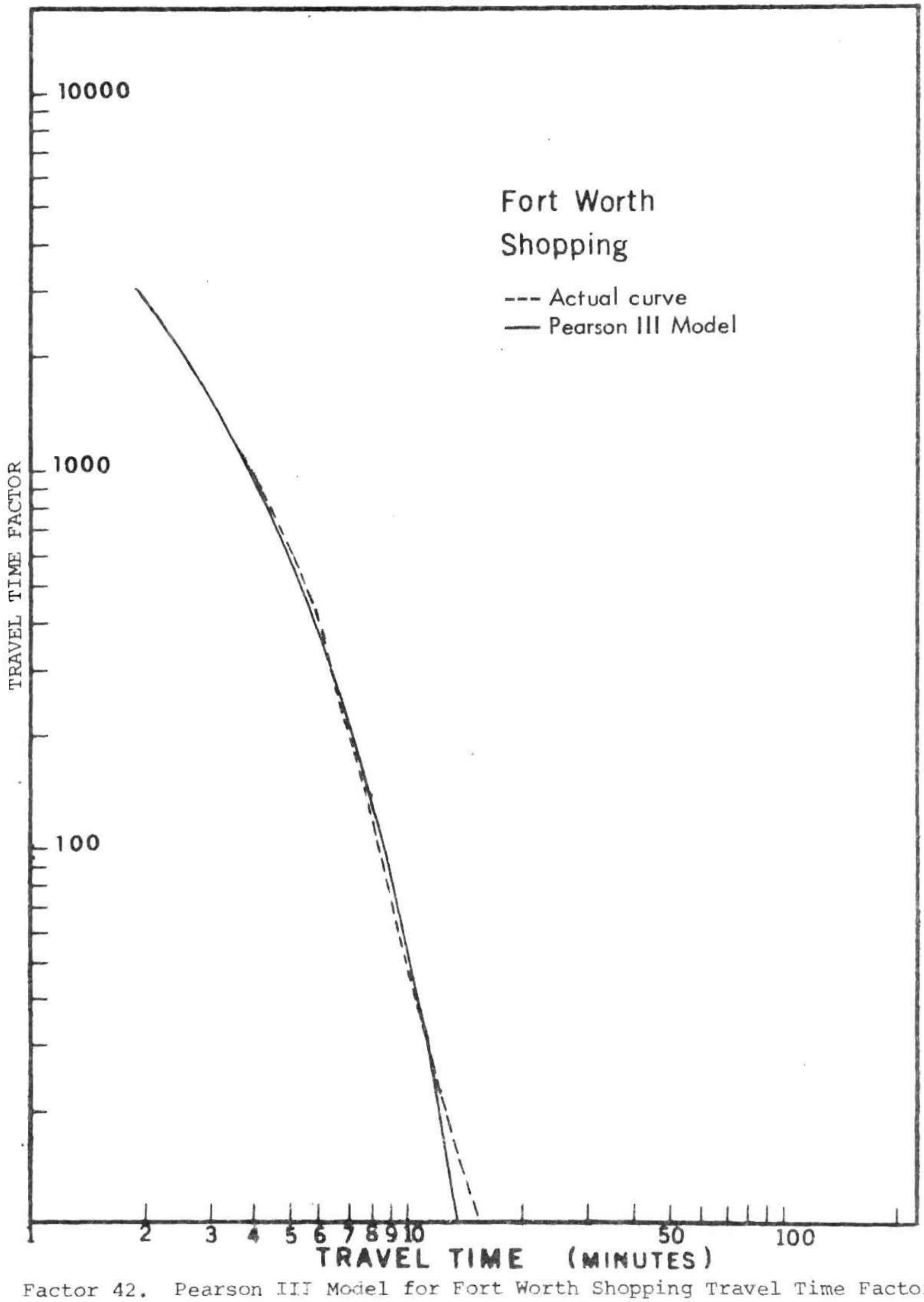


Figure 41. Pearson III Model for Providence Shopping Travel Time Factor



Factor 42. Pearson III Model for Fort Worth Shopping Travel Time Factor

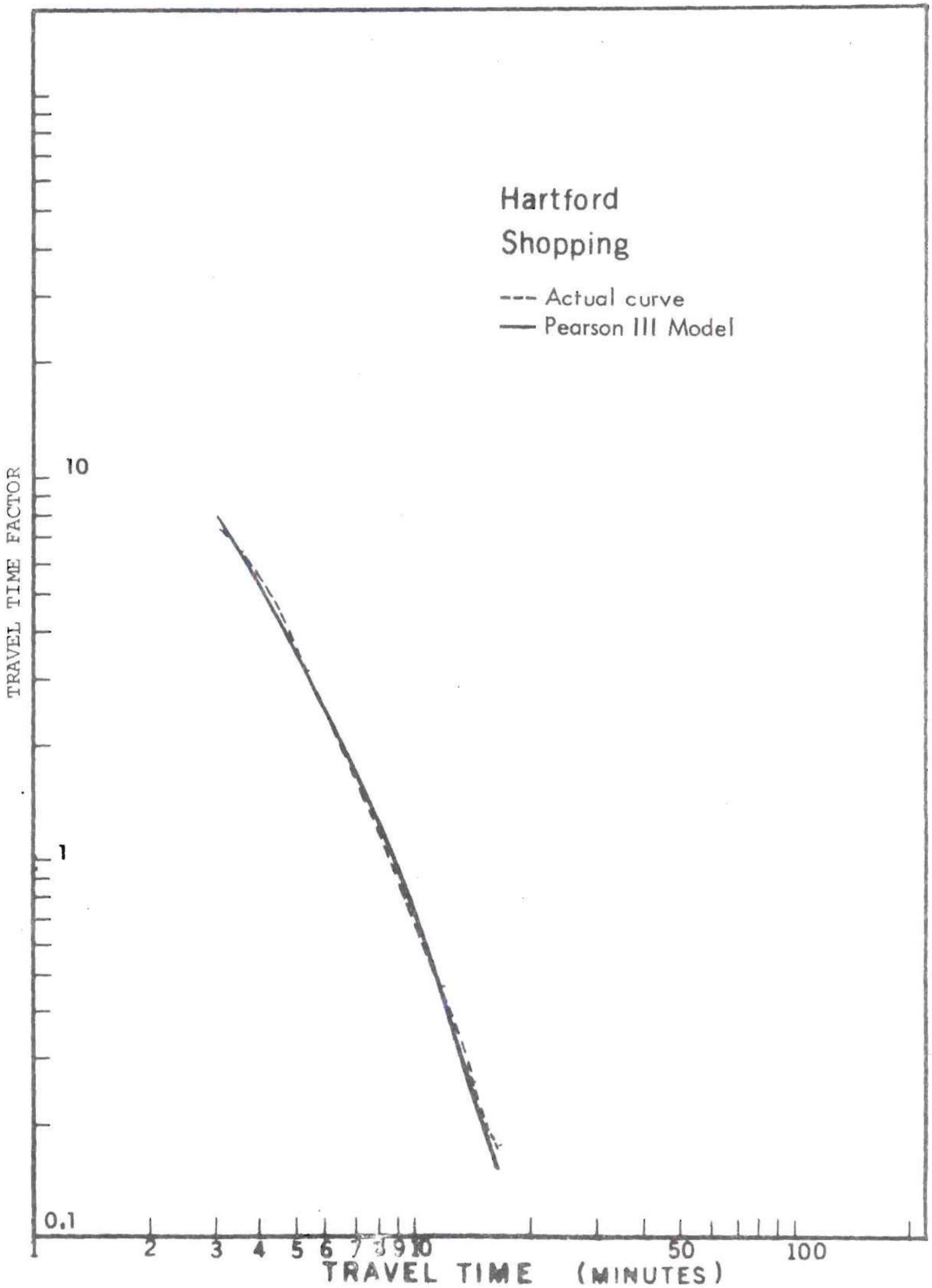


Figure 43. Pearson III Model for Hartford Shopping Travel Time Factor

Table 19. Linear Regression Analysis for A

Home Based Shopping Trips

Model

A = 12.74 x (Home Based Other Than Work Trips ÷ Total Trips)

	Observed A	Predicted A
Waterbury	-1.12	-1.61
Erie	-1.90	-1.61
Providence	-2.33	-1.85
Hartford	-4.27	-4.34
Fort Worth	-0.97	-1.16

Standard Error of Estimate = 0.45

Correlation Coefficient:

Home Based Other Than Work Trips ÷ Total Trips: r = 0.96

t-Value for Regression Coefficient
Home Based Other Than Work Trips ÷ Total Trips t = -5.64 (Significant at 2%)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	6.432	6.432	31.86	1%
About Regression	3	0.606	0.202		
Total	4	7.038			

Table 19A. Alternative Regression Analysis for A

Home Based Shopping Trips

Model

$$\ln |A| = 1.37 + 1.28 \times 10^{-3} \times \text{Total Trips per 1000 Population}$$

	Observed A	Predicted A
Waterbury	-1.12	0.95
Erie	-1.90	1.49
Providence	-2.33	2.56
Hartford	-4.27	3.35
Fort Worth	-0.97	1.69

Standard Error of Estimate = 0.394 (log transformed)
Correlation Coefficient:

Total Trips per 1000 Population: $r = 0.82$

t-Value for Regression Coefficient:
Total Trips per 1000 Population: $t = 2.48$ (Significant at 10% level)

Analysis of Variance for Regression

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	0.957	0.957	6.17	10%
About Regression	3	0.465	0.155		
Total	4	1.422			

Table 20. Linear Regression Analysis for p

Home Based Shopping Trips

Model

p = -1.92 + 0.0305 x (Home Based Other Than Work Trips ÷ Total Trips)

	Observed p	Predicted p
Waterbury	-0.35	-0.43
Erie	-0.45	-0.43
Providence	-0.49	-0.46
Hartford	-0.79	-0.79
Fort Worth	-0.39	-0.37

Standard Error of Estimate = 0.053

Correlation Coefficient:

Home Based Other Than Work Trips ÷ Total Trips: r = 0.97

t-Value for Regression Coefficient

Home Based Other Than Work Trips ÷ Total Trips:

t = 6.46 (Significant at 1%)

Analysis of Variance for Linear Regression

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	0.116	0.116	41.8	1%
About Regression	3	0.0083	0.0028		
Total	4	0.124			

Table 20A. Alternative Regression Analysis for p

Home Based Shopping Trips

Model

$$\ln |p| = 1.11 + 5.10 \times 10^{-7} \times \text{(Total Trips)}$$

	Observed p	Predicted p
Waterbury	-0.35	-0.36
Erie	-0.45	-0.43
Providence	-0.49	-0.50
Hartford	-0.79	-0.71
Fort Worth	-0.39	-0.48

Standard Error of Estimate = 0.18 (log transformed)

Correlation Coefficient: Total trips: r = 0.868

t-Value for Regression Coefficient:

Total Trips: t = 3.02 (Significant at 5%)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	0.297	0.297	9.13	10%
About Regression	3	0.098	0.033		
Total	4	0.395			

Table 21. Linear Regression Analysis for μ

Home Based Shopping Trips

Model

$$\mu = 8.06 - 15.79 \times \text{(Cars per Person)}$$

	Observed μ	Predicted μ
Waterbury	2.98	2.85
Erie	3.16	3.16
Providence	3.35	3.32
Hartford	1.99	1.91
Fort Worth	2.31	2.54

Standard Error of Estimate = 0.16

Correlation Coefficient: Cars per Person $r = 0.97$

t-Value for Regression Coefficient:

Cars per Person $t = -7.12$ (Significant at 1%)

Analysis of Variance for Linear Regression:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-Ratio	Level of Significance
Due to Regression	1	1.276	1.276	50.7	1%
About Regression	3	0.0755	0.0252		
Total	4				

APPENDIX D

RESULTS OF CORRELATION ANALYSIS

Table 22. Results of Correlation Analysis Correlation Coefficients Between Curve Parameters and Areawide Variables

Home Based Work

		m_1	m_2	A	c
1.	Total Study Area in Sq.Miles	-0.38	0.73	0.75	0.66
2.	Population	-0.41	0.69	0.74	0.33
3.	Population/Study Area	0.51	-0.71	-0.56	-0.60
4.	Employment	-0.40	0.69	0.74	0.32
5.	% Population Employment	0.52	-0.24	-0.07	-0.24
6.	Home Based Work Trips	-0.39	0.71	0.74	0.42
7.	Home Based Work Trips (% Total Trips)	-0.11	0.13	0.26	0.74
8.	Home Based Other Than Work Trips	-0.38	0.68	0.72	0.22
9.	Home Based Other Than Work Trips (% Total Trips)	-0.48	0.08	0.11	0.15
10.	Non Home Based Trips	-0.33	0.67	0.69	0.04
11.	Non Home Based Trips (% Total Trips)	0.28	-0.20	-0.27	-0.62
12.	All Trips	-0.36	0.69	0.72	0.25
13.	Home Based Work/1000 Population	0.75	-0.41	-0.29	0.77
14.	Home Based Work/Study Area	0.81	-0.73	-0.54	-0.63
15.	Home Based Other Than Work Trips/ 1000 Population	0.67	-0.47	-0.47	-0.74
16.	Home Based Other Than Work Trips/ Study Area	0.75	-0.81	-0.73	-0.80
17.	Non Home Based/1000 Population	0.33	0.05	0.02	-0.01
18.	Non Home Based/Study Area	0.70	-0.70	-0.69	-0.73
19.	All Trips/1000 Population	0.67	-0.28	-0.28	-0.36
20.	All Trips/Study Area	0.82	-0.79	0.68	-0.77
21.	Number of Cars	-0.37	0.70	0.73	0.37
22.	Cars/Person	-0.12	0.53	0.40	-0.10
23.	Persons/Car	-0.00	-0.44	-0.28	0.09
24.	Home Based Work Trips/Car	0.77	-0.57	-0.39	-0.33
25.	Home Based Other Than Work Trips/Car	0.74	-0.56	-0.53	-0.81
26.	Non Home Based Trips/Car	0.05	-0.32	-0.31	-0.68
27.	All Trips/Car	-0.37	0.72	0.09	-0.72

Table 23. Results of Correlation Analysis Correlation Coefficients Between Curve Parameters and Areawide Variables

Non Home Based Trips

		m_1	m_2	A	c
1.	Total Study Area in Square Miles	-0.16	0.91	-0.16	0.09
2.	Population	-0.21	0.79	-0.21	0.44
3.	Population/Study Area	0.09	-0.71	0.09	0.17
4.	Employment	-0.20	0.79	-0.20	0.42
5.	% Population Employment	0.21	-0.21	0.21	-0.40
6.	Home Based Work Trips	-0.20	0.84	-0.20	0.36
7.	Home Based Work Trips (% Total Trips)	0.42	0.55	0.43	0.18
8.	Home Based Other than Work Trips	0.18	0.70	-0.18	0.48
9.	Home Based Other than Work Trips (% Total Trips)	0.07	0.27	0.08	0.62
10.	Non Home Based Trips	-0.18	0.41	-0.18	0.11
11.	Non Home Based Trips (% Total Trips)	-0.48	-0.39	-0.48	-0.07
12.	All Trips	-0.18	0.69	-0.18	0.34
13.	Home Based Work/1000 Population	0.33	-0.53	0.33	-0.49
14.	Home Based Work/Study Area	0.26	-0.73	0.26	-0.15
15.	Home Based Other than Work Trips/ 1000 Population	0.09	-0.84	0.09	-0.21
16.	Home Based Other than Work Trips/ Study Area	0.15	-0.84	0.15	-0.01
17.	Non Home Based/1000 Population	-0.24	-0.00	-0.24	-0.31
18.	Non Home Based/Study Area	0.19	-0.76	0.20	-0.28
19.	All Trips/1000 Population	0.02	-0.50	0.02	-0.42
20.	All Trips/Study Area	0.21	-0.83	0.21	-0.16
21.	Number of Cars	-0.16	0.81	-0.16	0.42
22.	Cars/Person	-0.12	-0.24	-0.12	-0.27
23.	Persons/Car	0.04	0.29	0.04	0.37
24.	Home Based Work Trips/Car	0.37	-0.49	0.37	-0.41
25.	Home Based Other than Work Trips/Car	-0.18	-0.70	-0.18	0.11
26.	Non Home Based Trips/Car	0.49	-0.40	-.49	-0.71
27.	All Trips/Car	0.73	0.14	0.09	-0.50

Table 24. Results of Correlation Analysis Correlation Coefficients Between Curve Parameters and Areawide Variables

Shopping Trips

		A	p	u
1.	Total Study Area in Square Miles	-0.67	-0.63	-0.17
2.	Population	-0.56	-0.56	-0.20
3.	Population/Study Area	0.18	0.20	0.30
4.	Employment	-0.58	-0.56	-0.12
5.	% Population Employment	-0.18	-0.00	0.74
6.	Home Based Work Trips	-0.70	-0.70	-0.31
7.	Home Based Work Trips (% Total Trips)	0.40	0.54	0.88
8.	Home Based Other Than Work Trips	-0.67	-0.74	-0.63
9.	Home Based Other Than Work Trips (% Total Trips)	0.96	0.96	0.59
10.	Non Home Based Trips	-0.85	-0.93	-0.83
11.	Non Home Based Trips (% Total Trips)	-0.72	-0.83	-0.88
12.	All Trips	-0.81	-0.87	-0.69
13.	Home Based Work/1000 Population	-0.85	-0.84	-0.46
14.	Home Based Work/Study Area	0.13	0.14	0.28
15.	Home Based Other Than Work Trips/ 1000 Population	-0.23	-0.38	-0.79
16.	Home Based Other Than Work Trips/Study Area	0.18	0.16	0.18
17.	Non Home Based/1000 Population	-0.71	-0.66	-0.06
18.	Non Home Based/Study Area	-0.08	-0.12	-0.20
19.	All Trips/1000 Population	-0.83	-0.85	-0.40
20.	All Trips/Study Area	0.08	0.07	0.12
21.	Number of Cars	-0.69	-0.71	-0.41
22.	Cars/Person	-0.55	-0.68	-0.97
23.	Person/Car	0.56	0.69	0.94
24.	Home Based Work Trips/Car	-0.08	0.06	0.72
25.	Home Based Other Than Work Trips/Car	-0.24	-0.32	-0.33
26.	Non Home Based Trips/Car	0.00	-0.02	-0.30
27.	All Trips/Car	0.42	0.46	0.21

APPENDIX I

DERIVATION OF PEARSON I AND PEARSON III EQUATIONS

APPENDIX I

DERIVATION OF THE PEARSON I AND PEARSON III EQUATIONS (3)

Consider a differential equation of the form:

$$\frac{dy}{dx} = \frac{y(x+a)}{b_0 + b_1x + b_2x^2}$$

For certain conditions of the form of the denominator, if y = 0 then dy/dx = 0 and if x = -a, dy/dx = 0 and y ≠ 0, a maximum can exist at that point.

The equation may be written:

$$\frac{dy}{dx} \cdot \frac{1}{y} = \frac{x + a}{b_0 + b_1x + b_2x^2}$$

$$\frac{d(\ln.y)}{dx} = \frac{x + a}{b_0 + b_1x + b_2x^2} \qquad \cdots (1)$$

$$\frac{d(\ln.y)}{dx} = \frac{x + a}{b_2\left[x - \dfrac{-b_1 + \sqrt{b_1^2 - 4b_0b_2}}{2\ b_2}\right]\left[x - \dfrac{-b_1 - \sqrt{b_1^2 - 4b_0b_2}}{2\ b_2}\right]}$$

If the roots of $b_2x^2 + b_1x + b_0$ are real and different, then $4b_0b_2$ is positive, and $-b_1 + \sqrt{b_1^2 - 4b_0b_2}$ is positive while

$-b_1 - \sqrt{b_1^{\,2} - 4b_0b_2}$ is negative. This may also be verified by testing to see if $b_1^{\,2}/4b_0b_2$ is positive. This value will henceforth be called the curve criterion "k."

Therefore Equation (1) is of the form:

$$\frac{d(\ln.y)}{dx} = \frac{1}{b_2} \cdot \frac{x + a}{(x+A_1)(x-A_2)}$$

$$\frac{d(\ln.y)}{dx} = \frac{1}{b_2} \cdot \frac{A_1 - a}{A_1 + A_2} \cdot \frac{1}{x + A_1} + \frac{1}{b_2} \cdot \frac{A_2 + a}{A_1 + A_2} \cdot \frac{1}{x - A_2}$$

Integrating both sides of this equation:

$$\ln(y) = \frac{1}{b_2} \cdot \frac{A_1 - a}{A_1 + A_2} \cdot \ln(x+A_1) + \frac{1}{b_2} \cdot \frac{A_2 + a}{A_1 + A_2} \cdot \ln(x-A_2) + \text{Constant}$$

Putting the integration constant equal to y':

$$y = y' \cdot (x+A_1)^{\frac{1}{b_2} \cdot \frac{A_1-a}{A_1+A_2}} \cdot (x-A_2)^{\frac{1}{b_2} \cdot \frac{A_2+a}{A_1+A_2}}$$

In order to express the equation with origin at the mode, which is the most common form, for x + a write x:

$$y = y'[x + (A_1-a)]^{\frac{1}{b_2} \cdot \frac{A_1-a}{A_1+A_2}} \cdot [x - (A_2+a)]^{\frac{1}{b_2} \cdot \frac{A_2+a}{A_1+A_2}}$$

$$\therefore\ y = \left[\left[\frac{x}{(A_1-a)} + 1\right]\left[A_1-a\right]\right]^{\frac{1}{b_2}\cdot\frac{A_1-a}{A_1+A_2}} \cdot \left[\left[1 - \frac{x}{(A_2+a)}\right]\left[-A_2 - a\right]\right]^{\frac{1}{b_2}\cdot\frac{A_2+a}{A_1+A_2}}$$

$$y = y_0\left[\frac{x}{(A_1-a)} + 1\right]^{\frac{1}{b_2}\cdot\frac{A_1-a}{A_1+A_2}} \left[1 - \frac{x}{A_2 + a}\right] \qquad (2)$$

Let $\frac{1}{b_2}\cdot\frac{A_1 - a}{A_1 + A_2} = m_1$, then $a_1 = b_2(A_1+A_2)\cdot m_1$

Let $\frac{1}{b_2}\cdot\frac{A_2 + a}{A_1 + A_2} = m_2$, then $a_2 = b_2(A_1+A_2)\cdot m_2$

and $\frac{a_1}{a_2} = \frac{m_1}{m_2}$

Equation (2), expressed with origin at the mode may be written in the form:

$$y = y_0\left(1 + \frac{x}{a_1}\right)^{m_1}\cdot\left(1 - \frac{x}{a_2}\right)^{m_2} \qquad (3)$$

where $\frac{a_1}{m_1} = \frac{a_2}{m_2}$.

This is the form of the solution to the differential equation where the roots of $b_0 + b_1x + b_2x^2$ are real and different. This is the Pearson Type I curve which is the solution when the curve criterion k is small and negative.

In the case that $b_2 = 0$ in the expression $b_0 + b_1x + b_2x^2$, then Equation (1) reduces to:

$$\frac{dy}{dx} \cdot \frac{1}{y} = \frac{x + a}{b_0 + b_1x}$$

$$\therefore \ln.(y) = \int \frac{x + a}{b_0 + b_1x} dx$$

$$= \int \left(\frac{1}{b_1} + \frac{a - b_0/b_1}{b_1x + b_0}\right) dx$$

$$= \frac{x}{b_1} + (a-b_0/b_1) \cdot \ln(b_1x+b_0) + \text{Constant}$$

$$y = y'e^{x/b_1} (b_1x+b_0)^{(a-b_0/b_1)/b_1} \tag{4}$$

In order to express the equation in its usual form, with origin at the mode, x + a is replaced by x:

$$y = y' \ e^{(x-a)/b_1} [b_1(x-a) + b_0]^{(a-b_0/b_1)/b_1}$$

$$= y' \ e^{-a/b_1} \cdot e^{x/b_1} \cdot [b_1x + (b_0-ab_1)]^{(a-b_0/b_1)/b_1}$$

Let $\gamma = -\frac{1}{b_1}$, $\alpha = \frac{b_1}{b_0 - ab_1}$, and $y_o = y' e^{-a/b_1}$

Then the equation reduces to:

$$y = y_0 e^{-\gamma x}(1 + \frac{x}{\alpha})^{\gamma\alpha} \qquad (5)$$

This is a common form of the Pearson III distribution. The criteria for the occurrence of this equation is that $b_2 = 0$, or that $k = b_1^{\ 2}/4b_0b_2 = \infty$. In practice, the Pearson III distribution may be used as a model when k is large. In this study, adequate fit was obtained for $k > 10$.

APPENDIX II

CURVE FITTING BY METHOD OF MOMENTS FOR PEARSON I CURVES

APPENDIX II

CURVE FITTING BY THE METHOD OF MOMENTS FOR PEARSON I CURVES (3)

The equation of the Pearson I curve with origin at the mode is:

$$y = y_0\left(1 + \frac{x}{a_1}\right)^{m_1}\left(1 - \frac{x}{a_2}\right)^{m_2}$$

where $m_1/a_1 = m_2/a_2$.

Let $a_1 + a_2 = b$ and $z = \dfrac{a_1 + x}{a_1 + a_2}$.

Treating the ordinates as frequencies of occurrence, the area from the lower limit of the curve at $x = -a_1$ to the upper limit $x = a_2$ is the total frequency N.

$$N = \int_{-a_1}^{a_2} y_o\left(1 + \frac{x}{a_1}\right)^{m_1}\left(1 - \frac{x}{a_2}\right)^{m_2} dx$$

$$N = \int_{-a_1}^{a_2} \frac{y_o}{a_1^{m_1} \cdot a_2^{m_2}} \cdot (a_1+x)^{m_1} \cdot (a_2-x)^{m_2}\, dx$$

$$N = \int_0^1 \frac{y_o}{a_1^{m_1} \cdot a_2^{m_2}} [z(a_1+a_2)]^{m_1} \cdot [(1-z)(a_1+a_2)]^{m_2} (a_1+a_2)dz$$

$$N = \int_0^1 \frac{y_o(a_1+a_2)^{m_1+m_2+1}}{a_1^{m_1} \cdot a_2^{m_2}} \cdot z^{m_1}(1-z)^{m_2} \cdot dz$$

$$N = \frac{y_o(m_1+m_2)^{m_1+m_2} \cdot (a_1+a_2)}{m_1^{m_1} \cdot m_2^{m_2}} \cdot B(m_1+1,\ m_2+1)$$

$$\therefore\ y_o = \frac{N}{b} \cdot \frac{m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)}$$

Thus the full equation may be written in the following way:

$$y = \frac{N}{b} \cdot \frac{m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$$

In the curves observed for this study, m_1 in all cases was negative. In order to avoid calculations involving $m_1^{m_1}$, it was necessary to shift the origin of the curve to the beginning point. Calculation of equation form with shifted origin to the beginning of the curve:

$$y = \frac{N}{a_1+a_2} \cdot \frac{m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot \left(1 + \frac{x}{a_1}\right)^{m_1} \cdot \left(1 - \frac{x}{a_2}\right)^{m_2}$$

but $z = x + a_1$.

Substituting into the above equation:

$$y = \frac{N}{a_1+a_2} \cdot \frac{m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot \left(1 + \frac{z-a_1}{a_1}\right)^{m_1} \cdot \left(1 - \frac{z-a_1}{a_2}\right)^{m_2}$$

Multiplying through both numerator and denominator by $(a_1+a_2)^{m_1+m_2}$

$$y = \frac{N}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot$$

$$\frac{(a_1+a_2)^{m_1+m_2}\, m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \left(1 + \frac{z-a_1}{a_1}\right)^{m_1} \cdot \left(1 - \frac{z-a_1}{a_2}\right)^{m_2}$$

$$= \frac{N}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot$$

$$\frac{(a_1+a_2)^{m_1+m_2}\, m_1^{m_1} \cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \cdot \frac{z^{m_1}(a_1+a_2-z)^{m_2}}{a_1^{m_1} \cdot a_2^{m_2}}$$

$$= \frac{N}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\cdot\Gamma(m_2+1)} \cdot$$

$$z^{m_1}\cdot(a_1+a_2-z)^{m_2}\cdot\frac{(a_1+a_2)^{m_1+m_2}\cdot m_1^{m_1}\cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}\cdot a_1^{m_1}\cdot a_2^{m_2}}$$

but $\dfrac{m_1}{a_1} = \dfrac{m_2}{a_2}$.

$$\therefore \quad \frac{(a_1+a_2)^{m_1+m_2}\, m_1^{m_1}\cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}\, a_1^{m_1}\cdot a_2^{m_2}} = \frac{(a_1+a_2)^{m_1+m_2}\cdot m_1^{m_1}\cdot m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}\cdot\dfrac{m_1^{m_1}\cdot a_2^{m_1}}{m_2^{m_1}}\cdot a_2^{m_2}}$$

$$= \frac{(a_1+a_2)^{m_1+m_2}\, m_2^{m_1+m_2}}{(m_1+m_2)^{m_1+m_2}\, a_2^{m_1+m_2}}$$

$$= \left[\frac{(a_1+a_2)m_2}{a_2}\right]^{m_1+m_2}\cdot\frac{1}{(m_1+m_2)^{m_1+m_2}}$$

$$= (m_1+m_2)^{m_1+m_2}\cdot\frac{1}{(m_1+m_2)^{m_1+m_2}}$$

$$= 1$$

$$\therefore\ y = \frac{N}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\cdot\Gamma(m_2+1)} \cdot x^{m_1}\,(a_1+a_2-x)^{m_2}$$

where the origin is at the beginning of the curve. This may be expressed as

$$y = y'\ x^{m_1}(a_1+a_2-x)^{m_2}$$

where

$$y' = \frac{N}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\cdot\Gamma(m_2+1)}$$

This was the form used in the calculation of curve ordinates.

The Calculation of Moment Properties

The nth moment about the vertical line through $x = -a_1$, or the beginning of the curve may be expressed as

$$N\mu_n' = \int_{-a_1}^{a_2} \frac{y_o}{a_1^{m_1}\cdot a_2^{m_2}} \cdot (a_1+x)^n(a_1+x)^{m_1}(a_2-x)^{m_2}dx\ .$$

$$= \int_0^1 \frac{y_o(a_1+a_2)^{m_1+m_2+n+1}}{a_1^{m_1}\cdot a_2^{m_2}} \cdot z^{m_1+n}(1-z)^{m_2}dz$$

$$= \frac{y_o(m_1+m_2)^{m_1+m_2}\, b^{n+1}}{m_1^{\,m_1}\; m_2^{\,m_2}} \cdot \frac{\Gamma(m_1+n+1) \cdot \Gamma(m_2+1)}{\Gamma(m_1+m_2+n+2)}$$

Since $\Gamma(p) = (p-1)\,\Gamma(p-1)$

$$\mu_1{}' = \frac{b(m_1+1)}{m_1+m_2+2} \quad \text{First Moment}$$

$$\mu_2{}' = \frac{b^2(m_1+1)(m_1+2)}{(m_1+m_2+2)(m_1+m_2+3)} \quad \text{Second Moment}$$

Etc.

These moments are taken about the beginning of the curve. For calculation purposes it was necessary to calculate the forms of μ_2, μ_3, μ_4 about the mean. The change of axis calculation may be effected in the following manner:

If $\nu_n{}''$ is the nth moment about point B

$\nu_n{}'$ is the nth moment about point A

d is the distance from B to A

Then

$$\nu_n{}'' = \nu_n{}' - n\,d\,\nu_{n-1}' + \frac{n(n-1)}{2!}\,d^2\,\nu_{n-2}' - \frac{n(n-1)(n-2)}{3!}\,d^3\,\nu_{n-3}' + \cdots$$

Applying this to calculate μ_2, with $m_1{}' = m_1+1$, $m_2{}' = m_2+1$, $r = m_1{}'+m_2{}'$

$$\mu_2 = \mu_2' - n\ d\ \mu_1' + \frac{n(n-1)}{2!} d^2\mu$$

$$= \frac{b^2 m_1'(m_1'+1)}{r(r+1)} - \frac{2\ bm_1'}{r} \cdot \frac{bm_1'}{r} + \left(\frac{bm_1'}{r}\right)^2$$

$$= \frac{b^2\ m_1'm_2'}{r^2(r+1)}$$

Similarly:

$$\mu_3 = \frac{2\ b^3\ m_1'm_2'(m_2'-m_1')}{r^3(r+1)(r+2)}$$

$$\mu_4 = \frac{3\ b^4\ m_1'm_2'(m_1'm_2'(r-6) + 2\ r^2)}{r^4(r+1)(r+2)(r+3)}$$

Let $\beta_1 = \dfrac{\mu_3^{\ 2}}{\mu_2^{\ 3}}$, $\beta_2 = \dfrac{\mu_4}{\mu_2^{\ 2}}$ and $\varepsilon = m_1'm_2'$

then $\beta_1 = \dfrac{4(r^2-4\varepsilon)(r+1)}{\varepsilon(r+2)^2}$ or $\dfrac{\beta_1(r+2)^2}{4(r+1)} = \dfrac{r^2}{\varepsilon} - 4$

$\beta_2 = \dfrac{3(r+1)(2r^2+\varepsilon(r-6))}{\varepsilon(r+2)(r+3)}$ or $\dfrac{\beta_2(r+2)(r+3)}{3(r+1)} = \dfrac{2r^2}{\varepsilon} + r - 6$

Eliminating r^2/ε

$$r = \frac{6(\beta_2-\beta_1-1)}{3\beta_1-\beta_2+6}$$

Hence

$$\varepsilon = \frac{r^2}{4 + \frac{1}{4}\beta_1 \frac{(r+2)^2}{r+1}}$$

and from the equation for μ_2:

$$(a_1+a_2)^2 = b^2 = \frac{\mu_2(r+1)r^2}{\varepsilon}$$

Solving for m_1 and m_2 in terms of r and β, from $r = m_1' + m_2'$ and $\varepsilon = m_1' \cdot m_2'$

$$m_1 = \frac{1}{2}\left[r-2-r(r+2)\sqrt{\left(\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}\right)}\right]$$

$$m_2 = \frac{1}{2}\left[r-2+r(r+2)\sqrt{\left(\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}\right)}\right]$$

From the theory developed above, see Reference 3, the step-by-step solution of a curve fit by the method of moments can be enumerated.

1. $\mu_1 = \dfrac{\sum x_i f_i}{\sum f_i}$ where x is the abscissa and f is the frequency of occurrence or ordinate.

2. $s_i = x_i - \mu_i$ for all i

3. $$\mu_2 = \frac{\sum f_i s_i^{\,2}}{\sum f_i}$$

4. $$\mu_3 = \frac{\sum f_i s_i^{\,3}}{\sum f_i}$$

5. $$\mu_4 \quad \frac{\sum f_i s_i^{\,4}}{\sum f_i}$$

6. $$\beta_1 = \frac{\mu_3^{\,2}}{\mu_2^{\,3}}, \qquad \beta_2 = \frac{\mu_4}{\mu_2^{\,2}}$$

7. $$r = 6(\beta_2 - \beta_1 - 1)/(6 + 3\beta_1 - 2\beta_2)$$

8. $$A = a_1 + a_2 = \frac{1}{2}\sqrt{\mu_2}\ \sqrt{\beta_1(r+2)^2 + 16(r+1)}$$

9. $$m_1 = \frac{1}{2} \cdot \left(r-2-r(r+2)\ \sqrt{\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}}\right)$$

10. $$m_2 = \frac{1}{2} \cdot \left(r-2+r(r+2)\ \sqrt{\frac{\beta_1}{\beta_1(r+2)^2 + 16(r+1)}}\right)$$

11. $c = \mu_1 - a_1$

$$= \mu_1 - Am_1/(m_1+m_2)$$

Steps 8, 9, 10 and 11 result in the parameters for fitting the curve:

$$y = \frac{N}{A^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1) \cdot \Gamma(m_2+1)} \cdot (t-c)^{m_1}(A-t+c)^{m_2}$$

APPENDIX III

CURVE FITTING BY METHOD OF MOMENTS FOR THE PEARSON III CURVES

APPENDIX III

CURVE FITTING BY THE METHOD OF MOMENTS FOR THE PEARSON TYPE III CURVE (3)

The equation of the Pearson Type III distribution with origin at the mode is:

$$y = y_0 \left(1 + \frac{x}{a}\right)^{\gamma a} e^{-\gamma x}$$

Let $\gamma a = p$ and $z = \gamma(a+x)$, $dz = \gamma \cdot dx$.

Then if N is the total frequency

$$N = \int_{-a}^{\infty} y_0 \left(1 + \frac{x}{a}\right)^{p} e^{-\gamma x}\, dx$$

$$N = \int_{-a}^{\infty} y_o \frac{(a+x)^p}{a^p} \cdot e^{-\gamma x}\, dx$$

$$N = \int_{o}^{\infty} y_0 \left(\frac{z^p}{\gamma^p} \cdot \frac{1}{a^p}\right) e^{-(z-\gamma a)} \frac{1}{\gamma} \cdot dz$$

$$N = \int_{o}^{\infty} y_o\, z^p\, a^{-p}\, e^{-z+p}\, \gamma^{-(p+1)}\, dz$$

$$N = y_o \int_o^\infty z^p \, e^{-z} \, dz \cdot \frac{e^p}{\gamma \, p^p}$$

$$N = y_o \, \frac{ae^p}{p^{p+1}} \cdot \Gamma(p+1)$$

Hence $$y_o = \frac{N \cdot p^{p+1}}{a \, e^p \, \Gamma(p+1)}$$

The nth moment about the start of the curve (origin at the mode):

$$N \mu_n' = \int_{-a}^{\infty} y_o (1 + \frac{x}{a})^p \cdot e^{-\gamma x} \, (x+a)^n \, dx$$

$$= \frac{y_o \, e^p}{p^p \, \gamma^{n+1}} \int_o^\infty z^{p+n} \, e^{-z} \, dz$$

$$\mu_n' = \frac{y_o \, e^p}{N \, p^p \, \gamma^{n+1}} \cdot \Gamma(p+n+1)$$

$$= \frac{N \, p^{p+1}}{a \, e^p \, \Gamma(p+1)} \cdot \frac{e^p}{N \, p^p \, \gamma^{n+1}} \cdot \Gamma(p+n+1)$$

$$= \frac{\Gamma(p+n+1)}{\gamma^n \, \Gamma(p+1)}$$

Therefore,

$$\mu_1' = \frac{\Gamma(p+2)}{\gamma\ \Gamma(p+1)} = \frac{p+1}{\gamma}$$

$$\mu_2' = \frac{\Gamma(p+3)}{\gamma^2\ \Gamma(p+1)} = \frac{(p+1)(p+2)}{\gamma^2}$$

$$\mu_3' = \frac{\Gamma(p+4)}{\gamma\ \Gamma(p+1)} = \frac{(p+1)(p+2)(p+3)}{\gamma^3}$$

To calculate the moments about the mean, the expressions must be modified for a shift of $\frac{p+1}{\gamma}$. About the mean the moments are μ_0, μ_1, μ_2, etc. Since

$$\mu_n = \mu_n' - n\ d\ \mu_{n-1}' + \frac{n(n+1)}{2!} \cdot d^2\ \mu_{n-2}' - \cdots$$

where d is the amount of shift of origin.

Then

$$\mu_0 = 1 \quad \text{(Total Frequency of Unity)}$$

$$\mu_1 = 0 \quad \text{(Origin at the Mean)}$$

$$\mu_2 = \frac{(p+2)(p+1)}{\gamma^2} - 2\ \frac{(p+1)}{\gamma}\ \frac{(p+1)}{\gamma} + \left[\frac{(p+1)}{\gamma}\right]^2 \cdot 1 = \frac{p+1}{\gamma^2} \qquad (1)$$

Similarly

$$\mu_3 = \frac{2(p+1)}{\gamma^3} \tag{2}$$

Dividing (1) by (2),

$$\gamma = \frac{2\ \mu_2}{\mu_3}$$

Therefore
$$p = \frac{4\ \mu_2^{\ 3}}{\mu_3^{\ 2}} - 1$$

and
$$a = \frac{p}{\gamma} = \frac{2\ \mu_2^{\ 2}}{\mu_3} - \frac{\mu_3}{2\mu_2}$$

APPENDIX IV

METHOD OF CONVERGENCE FOR MODELLING

APPENDIX IV

METHOD OF CONVERGENCE FOR MODELLING

The method of moments described in Appendices II and III is a method of curve fitting whereby properties of a series of points are equated to similar properties of the Pearson I and Pearson III distributions, respectively. The fits achieved with the first fitting process in general were extremely close. A method was devised which improved the degree of fit obtained over the range of interest of the travel time curve by an iterative procedure programmed into the computer procedure.

As has been described, the lack of fit, D_t, of the model at the minute intervals utilized was defined in the following way:

$$D_t = \frac{(\text{Model Value}_t - \text{Travel Time Curve Value}_t)^2}{\text{Travel Time Curve Value}_t}$$

where the subscript t denotes the particular time interval.
In order that the total lack of fit throughout the range of interest should be weighted to reflect the importance of the trip length, each value D_t was multiplied by the weighting factor W_t, where W_t was the percentage of trips occurring at travel time t.

An estimate of the weighted total lack of fit was obtained by the summation of D_tW_t over the range of fitting.

Improvement in the total weighted fit was obtained by an iterative procedure which recalculated the modelling curve, based on the last model estimate and its residual error. If the travel time factor was Y_t and the model estimate was Ye_t, the residual at the time interval t was therefore $e_t = Y_t - Ye_t$.

A new Ye_t closer to Y_t could be obtained by a reiteration of the modelling process, but starting with $(Y_t + e_t)$ instead of Y_t, as shown in Figure 44. The modelling process was cyclic with each successive input being calculated from the previous cycle. At each iteration $\sum D_t W_t$ was calculated over the range of interest. It was found that this value, similar in character to a weighted Chi-Square value, passed through a minimum value during the course of the iteration cycles. The value was considered to converge satisfactorily at its minimum value provided that the change in parameters between the minimum cycle and the next was not greater than 5 per cent. Where the change was greater than 5 per cent, the process was rerun with successive values of Y_t being calculated from $(Y_t + k \cdot e_t)$ where k was less than unity. That was repeated until convergence under the above criteria was obtained.

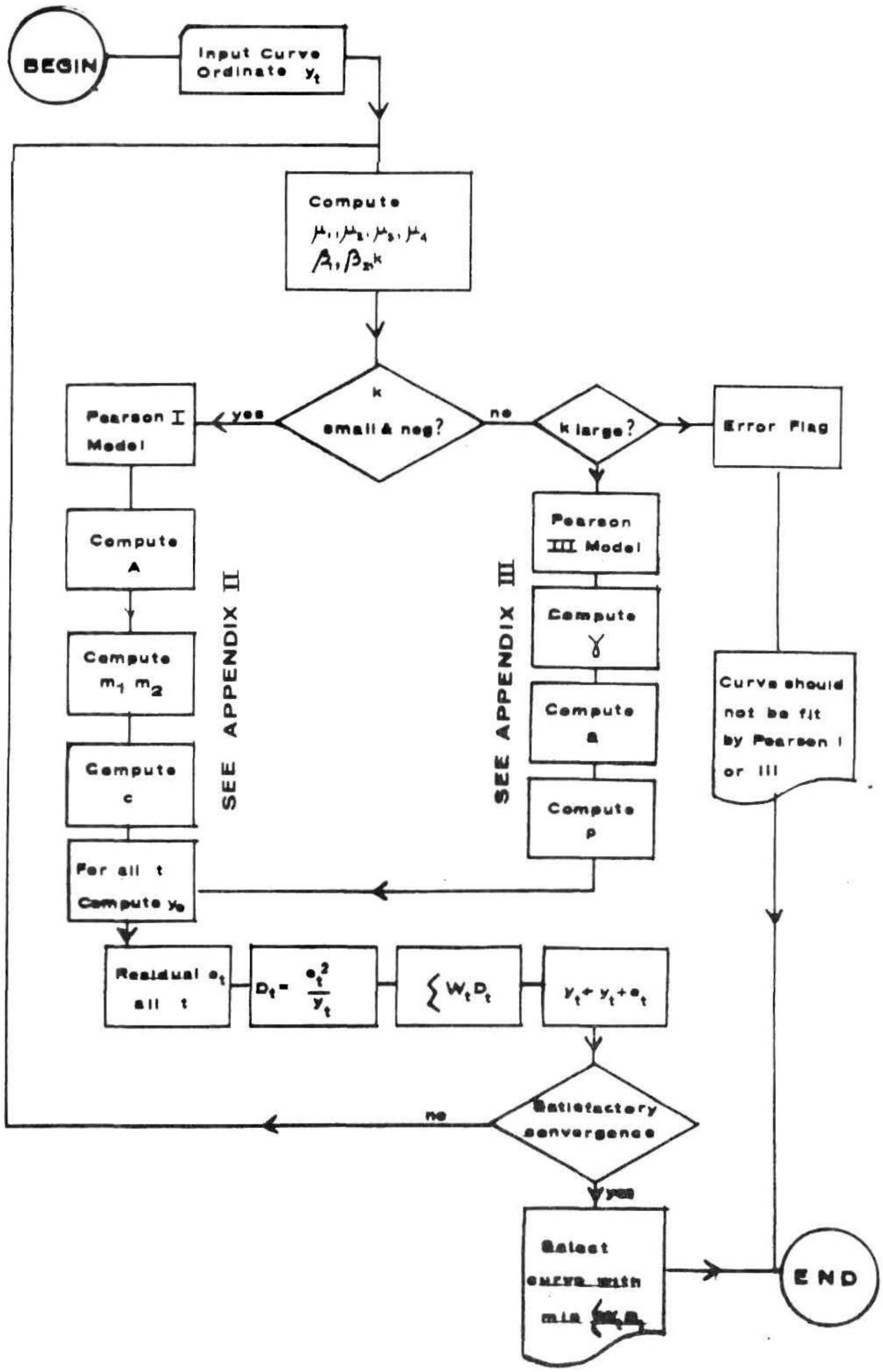


Figure 44. Flow Chart of the Modelling Process

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